EFFECT OF VOLUME FRACTION ON NON-NEWTONIAN FLOW OF DUSTY FLUID BETWEEN TWO OSCILLATING PARALLEL PLATES

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Abstract: In this paper the isothermal flow of dusty viscous incompressible conducting between two parallel plates is studied. The effect of various parameters associated with the flow problem have been analysed with the help of graphs. These results show that fluid velocity and particle velocity are effected by magnetic field for different values of volume fraction and non Newtonian factor.

Keywords: Incompressible fluid, skin fraction, magnetic field, non Newtonian factor, volume fraction.

Introduction: The study of heat and mass transfer in particular flows is emerging as an important topic of research. The study is very useful in scientific and technological applications such as sedimentation, purification of water and in petroleum industry etc.

The field is so important that number of scientists is motivated for research on the topic Dubey, and Srivastava (1972) investigated unsteady flow of dusty viscous flow with uniform distribution of dust particle in a channel bounded by two parallel flat plat. Prasad and Ramacharyulu N.C.P.studied unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient**.** Debnath and Ghosh (1989) studied unsteady hydro magnetic flows of a dusty viscous fluid between two oscillating plates. Sharma and Mathur (1995) investigated Steady Laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink. Suraju Olusegum Azadi (2005) studied unsteady flow of dusty viscous fluid between two parallel plates**.** HazemAttia (2005) investigated the topic of unsteady flow of dusty conducting fluid between parallel porous plates with temperature dependent viscosity.

Sreeharireddy, Nagarajan and Sivaiah (2009) studied MHD flow of a dusty viscous conducting liquid between two parallel plates. Khare and Singh(2010) investigated the MHD flow of a dusty viscous Incompressible fluid confined between two vertical walls with volume fraction of dust.

Saffman (1962) derived the equation describing the motion of gas carrying small dust particle and equations satisfied by small disturbances of a steady laminar flow.

Ganguly and Lehri studied the motion of an isothermal dusty viscous incompressible fluid between two infinite parallel plates where both plates are considered to be oscillating harmonically with different amplitudes and frequency in their own planes.

In another application Haldar investigated the flow of blood through a constricted artery in the presence of an external transverse magnetic field using Adomian's decomposition method .The expression for the two term approximation to the solution of stream function, axial velocity component and wall shear stress and obtained in this analysis.

The present paper considers the motion of an isothermal conducting dusty viscous incompressible fluid between two infinite parallel plates under the influence of gravitational precipitation, using analytical solutions.

Formulation of problem

Let us we consider two dimensional incompressible plane viscous flow between two parallel plates with distance d apart. We consider axial flow of fluid between the plates, as the plates are very wide and very long. For an isothermal flow the equations of motion of an unsteady dusty viscous incompressible fluid under gravity are

$$(1-\varphi)\frac{\partial U'}{\partial t'} + (U'.\nabla)U' = \frac{-1}{\rho}\nabla p - \frac{\sigma B_0^2}{\rho(1+c^2)} + v\frac{\partial^2 U'}{\partial y'^2} + \frac{KN}{\rho}(V'-U') + g' \quad (1)$$

$$\nabla.U' = 0 \quad (2)$$

$$\frac{\partial v\prime}{\partial t} + (V'.\nabla)V' = g' + \frac{K}{m'}(U'-V') \quad (3)$$

$$\frac{\partial N}{\partial t} + \nabla.(NV') = 0 \quad (4)$$

Where U' and V' are the local velocities of fluid and dust particles,

ρ : Density of fluid,

p : Static fluid pressure,

v : Kinetic viscosity,

N : Number of dust particles per unit volume

K : Resistances coefficient ($K = 6\pi\mu a$)

φ : Volume fraction of dust particle

c : Non Newtonian factor,

B_0: Constant external magnetic field parameter,

g' : Acceleration due to gravity and

m' : Mass of the particles.

From the equation of continuity we have

$$\nabla.U' = 0 \; i.e: \frac{\partial U\prime}{\partial x\prime} = 0 \qquad U' = U'(y') \quad (5)$$

$$\nabla.V' = 0 \; \text{i.e}: \frac{\partial V\prime}{\partial x\prime} = 0 \qquad V' = V'(y') \quad (6)$$

So equation (1) and (3) becomes

$$(1-\varphi)\frac{\partial U'}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} - \frac{\sigma B_0^2 U'}{\rho(1-c^2)} + v\frac{\partial^2 U'}{\partial y'^2} + \frac{KN}{\rho}(V'-U') + g' \quad (7)$$

$$\frac{\partial V'}{\partial t} = g' + \frac{K}{m}(U'-V') \quad (8)$$

Case I: Oscillating parallel plates

In this case we consider the parallel plates are oscillating. So let us we consider the lower plate is oscillating at an amplitude of a_1 and with frequencyλ_1, and the upper plate oscillates at an amplitude a_2 with frequency λ_2.Moreover we consider that the flow has a negligible convection ,have constant pressure gradient N .The boundary conditions in this case are

$$U' = a_1 e^{-i\lambda_1 t} \qquad at \qquad y' = 0$$

And

$$U' = a_2 e^{-i\lambda_2 t} \qquad at \qquad y' = d \quad (9)$$

Now eliminating v between (7) and (8) we get

$$(1-\varphi)U'_{tt} = V'U'_{yyt} + \frac{KV'}{m}U'_{yy} - \left\{(1-\varphi)\frac{K}{m} + \frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{KN}{\rho}\right\}U'_t - \frac{K\sigma B_0^2 U\prime}{\rho(1+c^2)} + \left\{\frac{Kg'}{m} + \frac{KNg'}{\rho} - \frac{K}{m\rho}\frac{dp}{dx}\right\} \quad (10)$$

Now since $\frac{dp}{dx} = (\rho + mN)g'$, so equation (10) becomes

$$(1-\varphi)U'_{tt} = V'U'_{yyt} + \frac{KV'}{m}U'_{yy} - \left\{(1-\varphi)\frac{K}{m} + \frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{KN}{\rho}\right\}U'_t - \frac{K\sigma B_0^2 U\prime}{\rho(1+c^2)} \quad (11)$$

Now from the boundary conditions we seek the solution as

$$U' = a_1 f(y)e^{-i\lambda_1 t} + a_2 g(y)e^{-i\lambda_2 t} \quad (12)$$

Consider $A = \frac{KV\prime}{m}$,

$B = (1-\varphi)\frac{K}{m} + \frac{\sigma B^2}{\rho(1+c^2)} + \frac{KN}{\rho}$ And

$C = \frac{K\sigma B^2}{\rho(1+c^2)}$

So equation (11) becomes

$$(1-\varphi)U'_{tt} = V'U'_{yyt} + AU'_{yy} - BU'_t - CU' \quad (13)$$

Now substituting the values of U'_{tt} , U'_{yyt}, U'_{yy} , U'_t and U' in (12) we get

$$f'' + Pf = 0 \quad (14)$$

Where $P = \frac{(1-\varphi)\lambda^2 + iB\lambda_1 - C}{A - iv\lambda_1}$

Similarly we can write

$$g'' + Qg = 0 \quad (15)$$

Where $Q = \frac{(1-\varphi)\lambda_2^2 + iB\lambda_2 - C}{A - iv\lambda_2}$

Now the solution of (14) can be written as

$$f(y) = C_0 \cos Py + C_1 \sin Py$$

With boundary conditions

$$f(y) = 1 \quad and \quad f(d) = 0$$

Using these boundary conditions we get

$$f(y) \frac{\sin P(d-y)}{sinPd} \quad (16)$$

Similarly the solution of (15) can be written as

$$g(y) = C_2 \cos Qy + C_3 \sin Qy$$

With boundary conditions $g(0) = 0 \quad and \quad g(d) = 1$ using these we get

$$g(y) = \frac{\sin Qy}{\sin Qd} \quad (17)$$

Using (16) and (17) in (12) we get

$$U' = \frac{a_1 \sin P(d-y)}{\sin Pd} e^{-i\lambda_1 t} + \frac{a_2 \sin Qy}{\sin Qd} e^{-i\lambda_2 t} \quad (18)$$

Substituting (18) in (7) we get

$$V' = \frac{mg\prime}{K} + \left(1 - \frac{i\lambda_1\rho(1-\varphi)}{KN} + \frac{vP^2\rho}{KN} + \frac{\sigma B_0^2}{(1-c^2)KN}\right) a_1 \frac{\sin P(d-y)}{sinPd} e^{-i\lambda_1 t} + \left(1 - \frac{i\lambda_2\rho(1-\varphi)}{KN} + \frac{vQ^2\rho\rho}{KN} + \frac{\sigma B_0^2}{(1+c^2)KN}\right) a_2 \frac{sinQy}{sinQd} e^{-i\lambda_2 t} \quad (19)$$

Case II: Fixed upper plates

In this case we consider that the upper plate is fixed and the lower plate executes simple harmonic motion in its own plane . So we have

$$a_2 = 0 \; and \; \lambda_2 = 0$$

So the velocity of the fluid becomes

$$U' = \frac{a_1 \sin P(d-y)}{\sin Pd} e^{-i\lambda_1 t} \quad (20)$$

And that of dust particle becomes

$$V' = \frac{mg\prime}{K} + \left(1 - \frac{i\lambda_1\rho(1-\varphi)}{KN} + \frac{vP^2\rho}{KN} + \frac{\sigma B_0^2}{(1-c^2)KN}\right) a_1 \frac{\sin P(d-y)}{sinPd} e^{-i\lambda_1 t} \quad (21)$$

Case III: Both plates oscillating with same amplitude and frequency

In this we consider the case where both the plates oscillates at an amplitude a

and frequency λ.

So we have $a_1 = a_2 = a \; and \; \lambda_1 = \lambda_2 = \lambda \; and \; P = Q = R$

$$U' = \frac{a \sin R(d-y)}{\sin Rd} e^{-i\lambda t} + \frac{a \sin Ry}{\sin Rd} e^{-i\lambda t}$$

Or

$$U' = \frac{a \cos R\left(\frac{d}{2} - y\right)}{\cos R\frac{d}{2}} e^{-i\lambda t} \quad (21)$$

And

$$V' = \frac{mg\prime}{K} + \left(1 - \frac{i\lambda\rho(1-\varphi)}{KN} + \frac{vR^2\rho}{KN} + \frac{\sigma B_0^2}{(1+c^2)KN}\right) a \frac{cosR\left(\frac{d}{2} - y\right)}{cos\frac{Rd}{2}} e^{-i\lambda t} \quad (22)$$

Table I: Magnetic field (B_0) and velocity of fluid (U')

$$K = 0.001, v = 0.01, m = 0.001, \sigma = 0.01, \rho = 0.01, a_1 = 0.3, a_2 = 0.4, \varphi = 0.01,$$

$$N = 0.1, \lambda_1 = 0.01, \lambda_2 = 0.02, d = 2, y = 1.1, t = 1\ g' = 1$$

B_0	U'(absolute)	U'(absolute)	U'(absolute)	U'(absolute)
	(non Newtonian factor)c =0.03	(non Newtonian factor)c =0.09	(non Newtonian factor)c =0.15	(non Newtonian factor)c =0.21
0.1	0.167776466	0.167536919	0.167068061	0.166390045
0.2	0.145622836	0.144836542	0.143387772	0.141538619
0.3	0.113415589	0.115247231	0.118441738	0.12242261
0.4	0.050268354	0.051201317	0.051660584	0.049105379
0.5	0.118338833	0.113799781	0.112159403	0.122897776
0.6	0.161351103	0.154770504	0.140764362	0.143381174
0.7	0.032558387	0.035672898	0.043437744	0.051004829
0.8	0.17147406	0.157046059	0.14932462	0.180630296
0.9	0.036527139	0.032703666	0.045088656	0.041374898
1	0.144581501	0.162311824	0.138042453	0.180600079

Table II: Magnetic field (B_0) and velocity of Dust particle (V')

$$K = 0.001, v = 0.01, m = 0.001, \sigma = 0.01, \rho = 0.01, a_1 = 0.3, a_2 = 0.4, \varphi = 0.01,$$

$$N = 0.1, \lambda_1 = 0.01, \lambda_2 = 0.02, d = 2, y = 1.1, t = 1\ g' = 1$$

B_0	V'(absolute)	V'(absolute)	V'(absolute)	V'(absolute)
	(non Newtonian factor)c =0.03	(non Newtonian factor)c =0.09	(non Newtonian factor)c =0.15	(non Newtonian factor)c =0.21
0.1	0.759639424	0.761324372	0.764554958	0.769075479
0.2	3.496798445	3.420754264	3.277073218	3.081198447
0.3	10.63578318	10.63735892	10.61315571	10.5441582
0.4	13.7228085	13.8483534	13.68709974	12.6126925
0.5	73.57234225	69.69285922	66.97725277	71.04008837
0.6	217.18386	205.555326	182.3353968	178.6764947
0.7	73.59751839	79.83929774	96.62528152	116.6727768
0.8	713.3735212	644.1400636	595.8227325	689.0168964
0.9	233.8555021	204.7941829	281.1387925	259.0857149
1	1466.25359	1620.565625	1340.134864	1676.405529

Figure I: Magnetic field (B_0) and velocity of fluid (U')

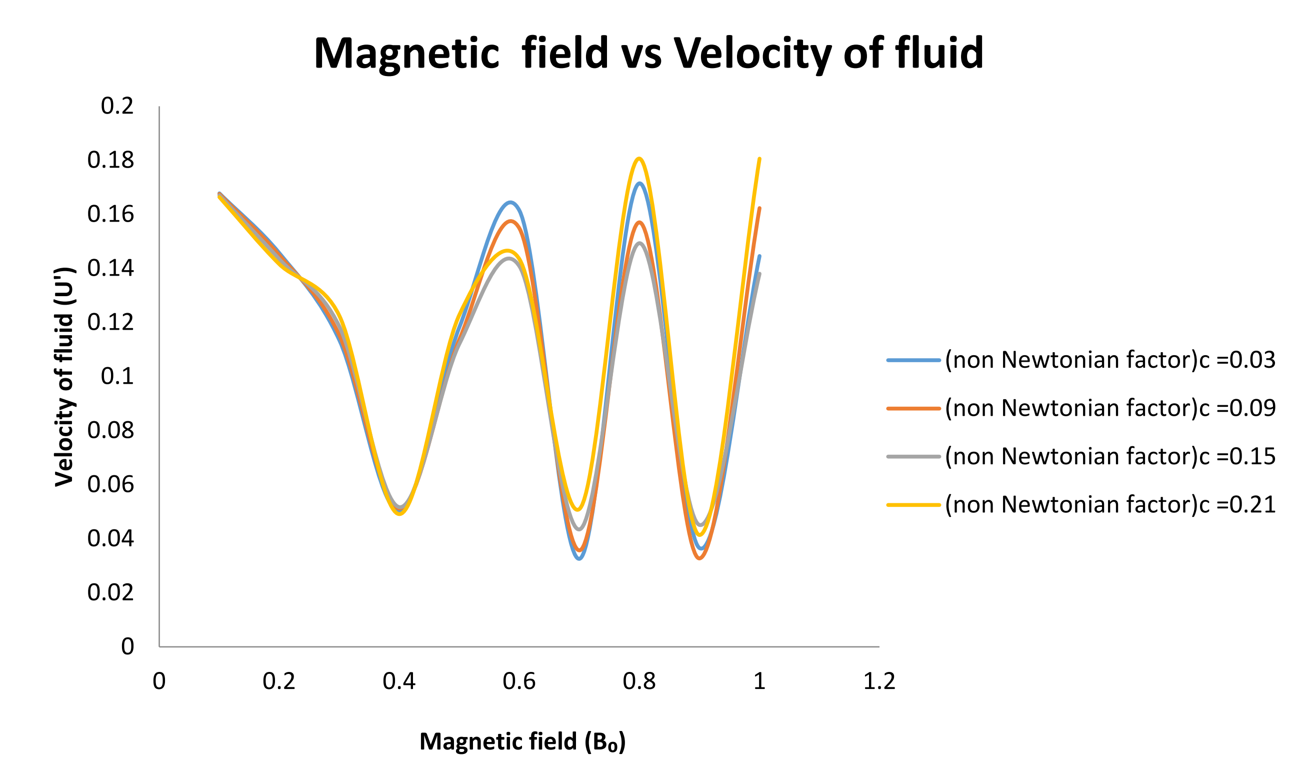
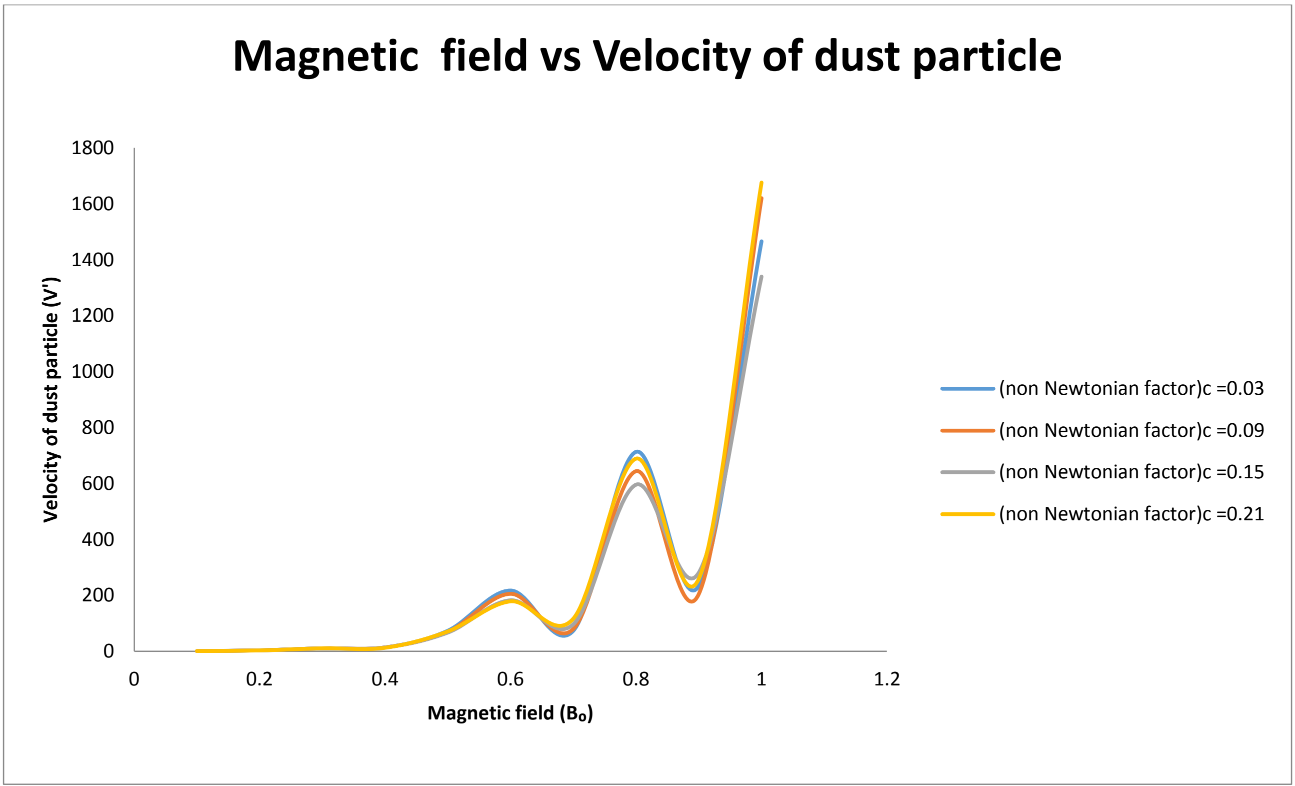


Figure II : Magnetic field (B_0) and Velocity of Dust particle(V')



Results and discussion

The above graph shows the variation in velocity of the fluid and the particle with change in the magnetic field parameter under the effect of volume fraction of the dust.

Figure I depict the change in the velocity of the fluid with changing magnetic field parameter. It can be shown from the graph that on increasing the value of magnetic field parameter the graph of the velocity takes wavy form .This is also clear from the expression of velocity of fluid obtained mathematically. The graph takes the resonance character having the same amplitude till the end. It has also been observed that on increasing the value of volume fraction of the dust the initial velocity also increases.

Figure II shows the change in the velocity of the dust with changing magnetic field of the system .It is evident mathematically and graphically that on increasing the magnetic effect the velocity takes the form of the wave. The waves so formed have very small amplitude in the beginning then the amplitude of the wave goes on increasing and finally becomes maximum at the end. It is also seen that on increasing the volume fraction of dust the initial velocity of the particle increases.

Thus we can conclude that the effect of volume fraction on the velocity of both the phases is same .It has been observed that the effect of magnetic field parameter is singnificant.The presence of magnetic field puts the system in wavy form with same amplitude till end, in case of fluid velocity, which is also evident mathematically as it contains sine and cosine terms. In case of particle velocity the graph also takes wavy form but with increasing amplitudes.

References

[1] **Attia, H. A. (2005):** "Unsteady flow of dusty conducting fluid between parallel porous plates with temperature dependent viscosity", Turkish Journal of Physics Vol 29, issue 4, pp.257-267.

[2] **Dubey, S.N. and Srivastava, L.P. (1972):** "Unsteady flow of dusty viscous flow with uniform distribution of dust particle in a channel bounded by two parallel flat plates" .Defence Science Journal. 7(22), 195-205.

[3] **Debnath L. and Ghosh A.K. (1989):** On unsteady hydro magnetic flows of a dusty viscous fluid between two oscillating plates, Journal of Applied Mathematics and Simulation, Vol. 2, No. 1, pp. 13-31.

[4] **Hadaman, M.H. and Barron, R.M. (1992):** "Analysis of the squeezing flow of dusty fluids", Applied Science and research, Vol.49, pp 345-354.

[5] **Khare, R. and Singh, N (2010)**: "MHD flow of a dusty viscous Incompressible fluid confined between two vertical walls with volume fraction of dust", Journal of International Academy of Physical Sciences, Vol.14 no: 2, pp 215-219.

[6] **Grubka, L. J. and Bobba, K. M (1985)**:"Heat Transfer Characteristics of a Continuous Stretching Surface with Variable Temperature," Journal of Heat Transfer, Vol. 107, No. 1, pp. 248-250.

[7] **Sharma ,P.R. and Singh , G.(2008):** " Unsteady MHD free Convective Flow and Heat Transfer Along a Vertical Porous Plate with variable suction and internal Heat Generation" , International Journal of Applied Mathematics and Mechanics , Vol. 4, No. 5 , pp. 1-8.

[8] **Prasad V.R. and Ramacharyulu N.C.P. (1979):** Unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient, Defence Science Journal, Vol. 30, pp. 125-130.

[9] **Saffman, P.G.,(1962)**: "On the Stability of Laminar flow of a Dusty Gas", Journal of Fluid Mechanics.,Vol 13,pp. 120-128.

[10] **Sharma, P.R.and Mathur, P. (1995):** Steady Laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink; Indian Journal Pure Applied Mathematics, 26(11), pp 1125.

[11] **Sreeharireddy, P., Nagarajan, A.S.and Sivaiah, M (2009): "**MHD flow of a dusty viscous conducting liquid between two parallel plates", Journal of Science and Research. I (2),(2009), pp.220-225.

[12] **Suraju Olusegum Azadi (2005):** A note on the unsteady flow of dusty viscous fluid between two parallel plates. Journal Applied Mathematics and Computing, Vol.18, No. 1-2, pp 393 – 403.

[13] **Sharma ,P.R. and Singh , G.(2008):** " Unsteady MHD free Convective Flow and Heat Transfer Along a Vertical Porous Plate with variable suction and internal Heat Generation" , International Journal of Applied Mathematics and Mechanics , Vol. 4, No. 5 , pp. 1-8.

[14] **Grubka, L. J. and Bobba, K. M (1985)**:"Heat Transfer Characteristics of a Continuous Stretching Surface with Variable Temperature," Journal of Heat Transfer, Vol. 107, No. 1, pp. 248-250.