International Journal of Emerging Technology and Innovative Engineering
Volume I, Issue 6, June 2015 (ISSN: 2394 – 6598)

THE DEVELOPMENT OF STRATEGIES HEURISTICS IN MATHEMATICAL PROBLEM SOLVING

Costică Lupu
Faculty of Sciences
Department of Mathematics-Informatics and Education Sciences
"Vasile Alecsandri" University of Bacău, România
Str. Mărășești 157, 600115, costica_lupu@yahoo.com

ABSTRACT
Composing and solving problems will challenge students throughout the entire school period as well as their entire life, but by being discreetly led towards discovering the solution, they will be enthusiastic and encouraged to obtain more and more performances.
Our study aims at elaborating a methodological model that may fully exploit heuristic didactic strategies in the heuristic solving of problems. The organization of the study will focus on improving the use and efficiency of heuristic mathematical techniques by relating to heuristic problem solving.
This article demonstrates the relevance of using heuristic problem-solving strategies in lessons of Mathematics, as a fundamental requirement with multiple valences in building thought operations, which leads to enhancing school performance.
The research was conducted during the 2014-2015 school year, involving two groups, each of them comprising 125 students: experimental group – the 8th grade from National Pedagogical College "Ștefan cel Mare", Bacău and 125 students: a control group – the 8th grade from "Miron Costin" Middle School, Bacău.
Various methods were applied during the research: conversation, experiment, analysis of activity products, the method of the tests, statistical processing of the data.

Keywords
Mathematics teaching, learning activity, problem solving, heuristic strategies

1. INTRODUCTION
In the case of solving problems, cognitive organizers may help the student, also in connection to cognitive training and other elements, guidelines for orienting thinking, exercises for operationalizing rules, to extract the relevant ideas from the cognitive structure regarding problem solving, the combination and processing of which may lead to the solving idea.
In solving a problem, some students may discover the solution or solving based on their own intuition, others may reach this performance relying on a minimal set of guidelines for orienting thought given through external conditions, which also plays a part in building their intuitive skills and reducing the effects of interference, inhibition, anxiety etc.
In solving a problem by resorting to intuition the students may also immediately discover the chain of implications that leads to demonstrating the relation. For other students, there is proposed a set of guidelines for orienting thought, through the application of which students may build trees for heuristic search, solving trees as we have called them. At first sight, this reminds of Gagné's hierarchies. But, at a closer look, it may be observed that passing from one level to another of the respective hierarchy implies resorting to subroutines (sets of operating schemes) amid restructuring and transforming ideas from the cognitive structure of the solver.
In solving problems, some students have the possibility to infer directly the structure of the solution (which is more difficult) or to resort to an analysis of the possible cases through dichotomy-trichotomy. This strategy was elaborated precisely based on a heuristic procedure, the principle of the excluded third element that lies at the basis of the problem analysis.
Hierarchizing the finalities of Mathematical training should be done on the basis of frame and reference objectives and the general and specific competences of teaching Mathematics in school. A more thorough analysis reveals nuances at the level of learning theories.
Thus, the representatives of unguided learning accentuate the process, not the product, training students through procedures that enable the acquisition of open systems of knowledge instead of a finite corpus of data. We would like to mention Bruner's approach (an advocate of semi-guided learning) who declared himself in favour of reduced teaching with a focus on the general principles of the learning content, accompanied by intense student activity in problem solving, with minimum guidance by the teacher in reaching rules and the reduction of the informational flow through problematization.
The analysis of the learning activity products, of their continuity and combination helps us identify the process of building a solving strategy at students. The examples of the ways in which – based on the elements of heuristic strategies, the organization of solving methods, cognitive organizers, guidelines for orienting thinking etc. – the students were determined and helped to form operating application schemes regarding the relation *solving strategy / observational indicators*,

support us in arguing that the cognitive scheme of a solving strategy is defined by the strategies (given by the guidelines for orienting thinking) that lead the student while performing the operations mentioned by the observational indicators.

The strategies heuristic nature of the guidelines for orienting thinking students, demonstrate the fact that there exists self-guidance in problem solving. This is consistent with orienting training towards self-training, the student being the artisan of his own construction, the subject being a source of actions and the object being the place on which they are practiced (the object – the contents of the symbolic universe: purely mental representations, signs or symbols that designate them).

2. RESEARCH DESCRIPTION

2.1 Research objectives

The research objectives were:

1. Knowledge of the heuristic teaching methods in order to be able to heuristically solve problems by studying the reference bibliography and the experience achieved during lessons of Mathematics;
2. Understanding the school syllabus for grades V-VIII;
3. Elaborating (initiating) a personal methodological process to fully exploit heuristic teaching strategies;
4. Organizing and conducting the experiment (in order to achieve the proposed objectives);
5. Analysing, processing and presenting the obtained results (in order to demonstrate, in an efficient way, the heuristic methods used in problem solving);
6. Formulating conclusions (in order to understand the efficiency of the experiment).

2.2 The research hypothesis

The organization of our experiment relied on the following **hypothesis**: If during the act of teaching-learning there are efficiently used heuristic mathematical problem-solving strategies, with multiple formative valences in building thought operations, then these will generate an increase in school performance and the students' results will be much improved.

2.3 Aspects of the research

The research was conducted during the 2014-2015 school year, involving two groups, each of them comprising 125 students: experimental group – the 8th A grade from National Pedagogical College "Ştefan cel Mare", Bacău, and 125 students : control group – the 8th B grade from "Miron Costin" Middle School, Bacău.

2.4 The stages of the experiment

The stage of initial evaluation aimed at observing the students' level of training by applying initial testing which consisted of observation protocols and a knowledge test (comprising different exercises and problems).

The stage of formative-ameliorative evaluation, during which there was introduced the progress factor and there were varied the manifestation circumstances by using active heuristic teaching methods, besides those used in the heuristic problem solving process.

The stage of final evaluation consisted in a comparison of the results obtained in the initial test, in order to highlight the students' progress/ regress at lessons of Mathematics, especially in problem solving.

The research variables are:

- the independent (introduced) variable, namely the use of active teaching methods;
- the dependent variable that leads to enhancing the efficiency of heuristic methods of solving problems and the students' school progress.

2.5 The research methodology

The research relied on the following knowledge methods and techniques**:**

1. The method of observation that is frequently used in school. Both spontaneous (passive) and scientific (generated) observation support the accumulation of a rich factual material, being able to provide data on the students' behaviour during lessons, breaks, extra-curricular and family activities.

2. The method of conversation was used to gather information from students, parents, the family's general practitioner or other acquaintances of the students. Thus, we could access data on the students' interests and aspirations, temperamental particularities, character features, general intelligence, family climate, material circumstances, daily regime, health, hobbies, likes/ dislikes in relation to certain activities, possibilities for doing homework.

3. The psychological analysis of the activity's results/ products provides information on several aspects related to the products of the activity. The data collected through this method was analysed by detaching appreciations and estimations related to the students' individuality, behaviour, inclinations and interests, the way in which they do their homework, their concern for correctness.

4. The method of tests represents a method that supports the diagnosis of the subject's development level – in this case, students – and formulating, on this basis, a prognosis regarding their evolution. *Docimological tests* provide quantitative information on the investigated phenomenon, when applied regularly during the instructive-educational process from the classes of Mathematics, as well as from other disciplines, have supported the determination of the level of knowledge, skills and the level of development of intellectual skills. They were conceived in relation with the established operational objectives, comprising sets of items meant to help us record and evaluate school performances.

5. The statistical-mathematical methods were used to analyse the obtained results that were inserted in analytical and synthetic tables, then systematized in centralized tables, graphs, histograms, circular diagrams, supporting the interpretation of data.

3. RESEARCH RESULTS AND DISCUSSIONS

3.1 Initial evaluation

During the observational stage, we applied an initial evaluation test. The test was elaborated by taking into account the objectives that had to be achieved by the end of the 4th grade, in order to establish the student's level of training.

Analysing the data from the tables, we may argue that:

- the results obtained by the students from the experimental class constitute information on the knowledge of the respective student, as well as the student's knowledge gaps;
- the total score at the level of the class represents the sum of the points obtained for each item plus one point from the office.

Following the recording of these data, our conclusions regarding the students' initial training level are the following: - the students had difficulties in solving problems; - the average of the experimental class is 7,3 , this representing the starting point in conducting our research.

The initial test was meant to establish the students' level of training. The test helped us notice the fact that the most difficult item was I_4 , whereas the best results were obtained at items I_1 , I_2 , I_3 . The data per student demonstrated relevant differences between the students who had solved 2-3 tasks and those who had solved all the tasks. We found that the level of the class is lower-intermediate.

Applying the initial test enabled us to identify the students' learning difficulties in the initial phase and, in relation to their extent, a more prolonged focus on the respective content until all the students have achieved a corresponding training level.

Analysing the graphs that represent the results obtained by the students from the *experimental class*, we found that from the 125 evaluated children, 54 obtained the mark VW (very well) representing 43%, 45 children obtained the mark W (well), representing 36%, and 26 children obtained the mark S (sufficient), representing 21% of the participants.

Analysing the graphs that show the results obtained by the students of the class, we found that in the initial evaluation, the results of the *control group* were the following: from the 125 evaluated children, 54 obtained the mark VW (very well), representing 43% of them, 37 children obtained the mark of W (well), representing 30%, and 20 children obtained the mark S (sufficient), representing 16%, whereas 14 children obtained the mark I (insufficient), representing 11 % of the participants.

Analysing the results obtained by the students with poorer results, we found that these are challenged by difficulties in solving the following tasks: - they do not perform calculi correctly; - they do not solve problems completely; - they do not compose problems following the given model; - they do not find the question that they need to raise in order to solve the problem.

Following the results obtained by the experimental class, we have noticed the fact that most students come across difficulties when solving problems.

3.2 Formative evaluation

The formative evaluation tests applied during lessons of Mathematics enabled the immediate knowledge of the students' learning difficulties. In order to eliminate the errors, we resorted to differentiating the activities. Following the analysis of the tests, there were presented the unachieved operational objectives, so that these may be aimed at during the proposed recovery activities.

Analysing the data, we may argue that although the students from the experimental class did not record major leaps in terms of their marks, almost all of them achieved better scores compared to the previous tests, therefore the learning experience was a success. We have also noticed the fact that the most frequent errors were those related to calculus, which indicates that the methods used in the heuristic solving of problems are known and acquired by the students of the class.

The formative evaluation tests applied during lessons of Mathematics enabled the immediate identification of errors and the students' learning difficulties. Looking at the tables with the data from the ameliorative formative tests and at the graphs with the scores and marks obtained in the initial tests, we may notice the fact that the school performance was improved as follows: - the average at the initial test for the experimental group was 7,3 and at formative test no. 1 the average was 8; - at formative test no. 2, there was a slight increase compared to the first test, the average being 8,2.

This increase is due to surpassing the more serious difficulties related to the contents of learning. The scores obtained were significantly higher than for the previous test. The results obtained highlight the relevance of formative evaluation tests applied during the learning activities and confirm the usefulness of the heuristic methods used. The fact that the results of the students from the experimental class were improved, with even the less industrious students achieving a promotion level, determined us to interfere, when it was necessary, with worksheets for repeating certain tasks, in order to achieve a more thorough acquisition of knowledge.

The progress obtained by the students compared to the initial test cannot be interpreted only as enhancement of percentages related to achieving objectives, but also in relation to the use of heuristic working methods, which led to activating the desire for performance or for increasing performance and, implicitly, a more active, conscious participation of students.

3.3 Summative (final) evaluation

On June the 1st 2014, there was applied the final evaluation of students through an evaluation test. In order to centralize and interpret the data, we have resorted to analytical and synthetic tables, frequency polygons, histograms and diagrams. The final evaluation test was designed in a similar manner to the initial one, so that the results obtained may be compared, the knowledge included in the syllabus being defined as operational objectives encoded as items.

The analysis of the analytical and synthetic tables of the histogram, the frequency polygon and the circular diagram revealed the fact that in the final evaluation, for the *experimental group*, the results were the following: from the 125 evaluated children, 80 obtained the mark VW (very well), representing 64%, 36 children obtained the mark W (well), representing 29 %, and 9 child obtained the mark S (sufficient), representing 7 % of the participants.

The analysis of the analytical and synthetic table, of the histogram, frequency polygon and circular diagram, revealed that in the final evaluation, the results for the *control group* were the following: from the 125 evaluated children, 55 obtained the mark VW (very well), representing 44%, 46 children obtained the mark W (well), representing 37%, whereas 14 children obtained the mark S (sufficient), representing 19% of the participants.

3.4 The comparative analysis of the data obtained in the initial and final evaluation form

In order to highlight the progresses related to improving relations following the conducted experiment and the applied methodology, we have proceeded to performing a comparative analysis of the two series from the initial and final evaluation.

Table 1. Comparative analysis for the experimental group

MARKS	Initial evaluation	Final evaluation
VERY WELL	54	80
WELL	45	36
SUFFICIENT	26	9
INSUFFICIENT	0	0

The comparison of the results obtained in the predictive and final test have revealed the fact that throughout the school year, as a result of the systematic application of active methods and differentiated learning during lessons, the progress of students was both qualitative and quantitative. This fact was easily seen in the ease and pleasure with which the students acquired a great amount of knowledge, with which they operated in solving problems and problem-situations (knowledge acquired especially through their personal effort), in the pleasure with which they worked throughout the entire school year.

The comparative analysis of the table and frequency polygon revealed the progress recorded at the end of the experiment by the experimental group. The results obtained in the final evaluation show an obvious difference from the scores obtained in the initial evaluation. This reveals the fact that the formative stage was efficient, the results obtained demonstrating the improvement of the results.

Table 2. Comparative analysis for the control group

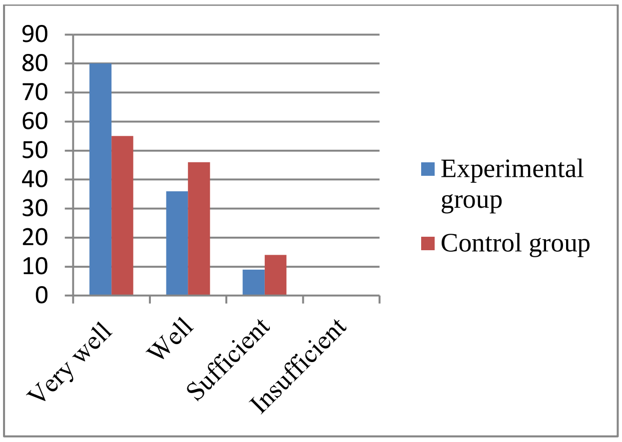
MARKS	Initial evaluation	Final evaluation
WERY WELL	54	55
WELL	37	46
SUFFICIENT	20	14
INSUFFICIENT	14	0

The comparative analysis of the table and frequency polygon reveals, for the control group, the fact that the number of students who obtained the mark VW remained the same, the number of those who obtained the mark W increased, the number of those with mark S did not increase but there increased the percentage for mark I. The results obtained in the final evaluation test did not increase significantly compared to the points obtained at the stage of initial evaluation.

Table 3. Comparative analysis between the two groups in the final evaluation

MARKS	EXPERIMENTAL GROUP	CONTROL GROUP
VERY WELL	80	55
WELL	36	46
SUFFICIENT	9	14
INSUFFICIENT	0	0

Figure 2. Frequency polygon comparative analysis for final evaluation



The comparative analysis of the histogram and frequency polygon reveals the progress recorded at the end of the experiment by the experimental group.

Calculating the average between the two tests (initial and final) and drawing a comparison between the two groups, there may be observed an increase in the school performance for the experimental group as compared to the control group.

4. CONCLUSIONS

In general, it may be said that solving problems constitutes the most appropriate way for achieving the objectives of teaching-learning Mathematics. The activity of Mathematics requires effort, focus and activation of all the components of the human psychic, particularly thought and intelligence. The intellectual effort put into composing and solving problems is, essentially, a continuous exercise that results in building the students' imagination and creativity.

From the instructive-educational perspective, solving problems constitutes the application of acquired knowledge in relation to mathematical operations and their properties, deepening and consolidating knowledge. In terms of practice, solving problems represents the seizing and understanding of the relations between sizes that we come across on a daily basis, for the solving of which it is not enough to know only the calculus technique.

The main objective of each lesson should serve not just training, but also education, an action where the leading role belongs to the educator. This should avoid the formal nature of the lesson and ensure an atmosphere of constant communication, the students participating with their own ideas, questions that the educator should tactfully guide towards the proposed educational goal. At the same time, he should aim at the accessibility of learning by challenging the student, in a systematic, conscious, gradated way, with obstacles that the student may overcome under his guidance.

The results obtained by applying the tests have generated the following findings: - the data obtained highlighted the higher results from the final test compared to the initial test, demonstrating the efficiency of the systematic mental training in finding several alternatives for solving a problem;
- the continuous, sustained solving of problems also helped the students with poorer results, removing their fear of failure and shyness; - the systematic training of students in finding as many possible alternatives for solving a problem leads to building the students' creativity; - involving the students in creative, active-participative activities gives the teacher the possibility to know individual particularities better, the style of each student, intelligence, will, temperament, behaviour, in a word, personality.

REFERENCE

[1] Aebli, H. (1998), *Zwölf Grundformen des Lehrens.Eine allgemeine Didaktik auf Kognitionspsychologischer Grundlage(Twelve basic forms of teaching.An approach to General Didactics founded on Cognitive Psyhology;1st.ed.:1983), 10th.ed. Stuttgart:Klett-Cotta.*

[2] Ausubel D.P., Robinson F.G. (1981) *Learning in the school. An introduction to the pedagogical psychology* (*in Romanian)*, Didactic and Pedagogical Publishing House, Bucharest, Romania.

[3] Gagne R.M. (1975), *Learning conditions*, *(in Romanian),* Didactic and Pedagogical Publishing House, Bucharest, Romania.

[4] Gagne R.M., Briggs L.J. (1977), *Principles of design training*, Didactic and Pedagogical Pubishing House, Bucharest, Romania.

[5] Lupu, C. (2006), *Teaching Mathematics*, *(in Romanian)*,Caba Publishing House, Bucharest, Romania.

[6] Lupu, C., Săvulescu D. (2000), *Teaching geometry*, (*in Romanian)*, Paralela 45 Publishing House, Pitesti, Romania.

[7] Lupu, C. (2013), *Establishment of Cognitive Functions trough Mathematical Education*, Quality and Efficiency in E-Learning, Vol 1 Book Series: eLearning and Software for Education, pages: 178-183.

[8] Postolică V., Lupu C. – *Euclidean Geometry and Computers*. Published online, as Original Research Article in the International Journal of Applied Science and Mathematics (IJASM), Vol. 2, Issue 1, Jan. – Feb., 2015, p. 1 – 6, ISSN (Online):2394 – 2894.