# THREE DIMENSIONAL UNSTEADY MIXED CONVECTIVE FLOW AND MASS TRANSFER PAST AN OSCILLATORY MOVING INFINITE VERTICAL POROUS PLATE IN THE PRESENCE OF HEAT SOURCE AND PERIODIC SUCTION

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## ABSTRACT

In this paper, three dimensional unsteady laminar mixed convective flow of a viscous, incompressible fluid and mass transfer past an oscillatory moving vertical infinite porous plate in the presence of heat source, is investigated when the suction at the plate is transverse sinusoidal and the plate temperature is periodic. The dimensionless governing equations are solved using a regular perturbation method. A parametric study is performed to illustrate the influence of physical parameters on velocity, temperature and concentration profiles and presented graphically. Also, the skin-friction coefficient, Nusselt number and Sherwood number at the plate are computed and numerical values are presented through tables.

**Key Words :** unsteady, heat source, mass transfer, Sherwood Number, porous plate

## INTRODUCTION

Simultaneous heat and mass transfer from different geometries embedded in porous medium has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery , packed-bed catalytic ,cooling of nuclear reactors and underground energy transport.

Lots of investigations have been carried out in two dimensional flows eg. Pai (1956), Bansal (1997), Schlichting and Gersten (2000) etc. if variation in the suction velocity distribution is transverse to the potential flow, then the situation of the flow field will convert to three – dimensional flow. Gersten and Gross (1974) studied the effect of a transverse sinusoidal suction velocity distribution on flow and heat transfer over a plane wall. Three dimensional fluctuating flow and heat transfer along a plate with suction was discussed by Singh et al. (1978). Singh (1993) presented three -dimensional viscous flow and heat transfer along a porous plate. Raptis and Masslas (1998) studied unsteady magnetohydrodynamic convection in a gray, absorbing – emitting but non-scattering fluid regime using the Rosseland radiation model. Unsteady MHD convective heat transfer past a semi- infinite vertical porous moving plate with variable suction has been studied by Kim (2000). Sharma et al. (2002) investigated three-dimensional viscous flow and heat transfer along porous plate in the presence of sinusoidal suction and wall temperature. Chamkha (2003) investigated steady flow, heat and mass transfer of electrically conducting fluid past a moving vertical surface in the presence of first order of the chemical reaction. Mankinde (2005) studied the natural convection heat and mass transfer in a gray, absorbing emitting fluid along a porous vertical translating plate. Mankinde et al. (2008) analyzed the effect of thermal radiation on the heat and mass transfer flow of a variable viscous fluid past a vertical porous plate permeated by a transverse magnetic field. Patil and Kulkarni (2008) considered the effects of chemical reaction on free convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Ibrahim et al. (2008) analyzed heat and mass transfer of a chemical convective process assuming an exponentially decreasing suction velocity at the surface of a porous plate. Sharma and Katta (2011) discussed mass transfer effect on unsteady mixed convective flow and heat transfer along an infinite vertical plate bounded with porous medium. Analytically solution for the effect of chemical reaction on the unsteady free convection flow past an infinite vertical permeable moving plate with variable temperature has been studied by Fayza Mohammed (2012). The unsteady coupled heat and mass transfer of two - dimensional MHD fluid over a moving oscillatory stretching surface with Soret and Dufour effects has been analyzed by Zheng et al. (2013).

Aim of this paper is to investigate three – dimensional unsteady flow heat and mass transfer past an oscillatory moving infinite vertical porous plate in the presence of heat source and periodic suction.

## FORMULATION OF THE PROBLEM

Consider an unsteady three – dimensional laminar flow of a viscous, incompressible fluid past an oscillatory moving vertical plate in the presence of heat source, thermal and concentration buoyancy effects and periodic plate temperature. The plate is assumed to be moving in $x^* - z^*$ plane such that $x^* - axis$ is oriented in the direction of the flow and $y^* - axis$ is perpendicular to the plane of the plate and directed into the fluid flowing laminarly with uniform free stream velocity.

The concentration level of the foreign mass presented has been considered to be very small. Since the plate is considered infinite in the $x^*$ direction, hence all physical quantities will be independent of $x^*$, however , the flow remains three dimensional due to variation of suction velocity applied at the plate of the form

$$v^*\left(z^*\right) = -V_0\left(1+\varepsilon\cos\frac{\pi V_0 z^*}{\upsilon}\right), \qquad \ldots(1)$$

where $V_0\left(>0\right)$ is cross – flow velocity, $\varepsilon\left(<<1\right)$ is small parameter and $\upsilon$ is the Kinematic viscosity. The negative sign indicates the suction through the plate .

Thus under the usual Boussinesq approximation the flow is governed by the following equations

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \qquad \ldots(2)$$

$$\rho\left[\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*}\right] = g\left(\rho_\infty - \rho\right) + \mu\left[\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right], \quad \ldots(3)$$

$$\rho\left[\frac{\partial v^*}{\partial t^*} + v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*}\right] = -\frac{\partial p^*}{\partial y^*} + \mu\left[\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right], \qquad \ldots(4)$$

$$\rho\left[\frac{\partial w^*}{\partial t^*} + v^*\frac{\partial w^*}{\partial y^*} + w^*\frac{\partial w^*}{\partial z^*}\right] = -\frac{\partial p^*}{\partial z^*} + \mu\left[\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right], \quad \ldots(5)$$

$$\rho C_p\left[\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} + w^*\frac{\partial T^*}{\partial z^*}\right] = \kappa\left[\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right] + \mu\phi^* + Q\left(T^* - T_\infty\right), \qquad \ldots(6)$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}+w^*\frac{\partial C^*}{\partial z^*}=D\left[\frac{\partial^2 C^*}{\partial y^{*2}}+\frac{\partial^2 C^*}{\partial z^{*2}}\right], \qquad ...(7)$$

where $\phi^*=2\left\{\left(\frac{\partial v^*}{\partial y^*}\right)^2+\left(\frac{\partial w^*}{\partial z^*}\right)^2\right\}+\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2+\left(\frac{\partial w^*}{\partial y^*}+\frac{\partial v^*}{\partial z^*}\right)^2+\left(\frac{\partial u^*}{\partial z^*}\right)\right\}$ is the viscous dissipation function, $u^*, v^*$ *and* $w^*$ are velocity components along $x^*, y^*$ *and* $z^*$ directions, respectively, $t^*$ is the time , g is the acceleration due to gravity, $\rho$ is the density of the fluid in the boundary layer and $\rho_\infty$ is density of fluid in the free stream, $\mu$ is the coefficient of viscosity , $p^*$ is the pressure, $C_p$ is the specific heat at the constant pressure, $\kappa$ is the thermal conductivity, $Q$ is the volumetric rate of heat generation / absorption and D is the diffusion coefficient.

The boundary conditions are

$$y^*=0\ :\quad u^*=U_w\left(1+\varepsilon e^{i\omega^* t^*}\right), v^*=-V_0\left(1+\varepsilon\cos\frac{\pi V_0 z^*}{\upsilon}\right)$$

$$w^*=0,\ T^*=T+\varepsilon\left(T_w-T_\infty\right)e^{i\omega^* t^*},\ C^*=C_w,$$

$$y^*\to\infty:\qquad u^*=U_\infty,\ v^*=-V_0,\ w^*=0,$$

$$p^*=p_\infty,\quad T^*=T_\infty,\ C^*=C_\infty. \qquad \ldots(8)$$

where $U_w, \mathrm{T}_w$ *and* $C_w$ denote mean velocity, temperature at the plate and concentration near the plate; $U_\infty, \mathrm{T}_\infty$ *and* $C_\infty$ are the free stream velocity, pressure, temperature and concentration of fluid , respectively.

For $B\Delta T<<1$ and $B^*\Delta C<<1, \left(\rho_\infty-\rho\right)$ can be expressed in terms of $\left(T^*-T_\infty\right)$ and $\left(C^*-C_\infty\right)$ as

$$g\left(\rho_\infty-\rho\right)=g\beta\rho\left(T^*-T_\infty\right)+g\beta\rho\left(C^*-C_\infty\right) \qquad \ldots(9)$$

## METHOD OF SOLUTION

Introducing the following dimensionless quantities

$$y=\frac{V_0 y^*}{\upsilon},\ z=\frac{V_0 z^*}{\upsilon},\ t=\frac{V_0^2 t^*}{4\upsilon},\ \omega=\frac{4\upsilon\omega^*}{V_0^2},\ u=\frac{u^*}{U_\infty},\ v=\frac{v^*}{V_0},\ \mathrm{Pr}=\frac{\mu C_p}{\kappa}, w=\frac{w^*}{V_0}$$

$$Sc=\frac{\upsilon}{D},\ \alpha=\frac{U_w}{U_\infty},\ Gr=\frac{\upsilon g\beta\left(T_w-T_\infty\right)}{U_\infty V_0^2}, Gm=\frac{\upsilon g\beta^*\left(C_w-C_\infty\right)}{U_\infty V_0^2},\ Ec=\frac{U_\infty^2}{C_p\left(T_w-T_\infty\right)},$$

$$\theta=\frac{\left(T^*-T_\infty\right)}{\left(T_w-T_\infty\right)}, \bar{\lambda}=\frac{V_0}{U_\infty},\ C=\frac{\left(C^*-C_\infty\right)}{\left(C_w-C_\infty\right)},\ p=\frac{p^*}{\rho V_0^2},\ S=\frac{\upsilon^2}{\kappa V_0^2}Q \qquad \ldots(10)$$

into the equations (2) to (7), we get

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \qquad \ldots(11)$$

$$\frac{1}{4}\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Gr\theta+GmC+\left[\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right], \qquad \ldots(12)$$

$$\frac{1}{4}\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\left[\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right], \qquad \ldots(13)$$

$$\frac{1}{4}\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\left[\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right], \qquad \ldots(14)$$

$$\Pr\left[\frac{1}{4}\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}\right]=\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}+\Pr Ec\phi+S\theta, \qquad \ldots(15)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial y}=\frac{1}{Sc}\left[\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right], \qquad \ldots(16)$$

where $\phi=2\bar{\lambda}^2\left\{\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2\bar{\lambda}^2+\left(\frac{\partial u}{\partial z}\right)^2\right\}$, $Gr$ is Grashof number due to temperature difference, $Gm$ is modified Grashof number due to concentration difference, $\Pr$ is Prandtl number, $Sc$ is the Schmidt number, $Ec$ is the Eckert number and $S$ is the rate of heat generation parameter.

The corresponding boundary conditions are reduced to

$$\begin{aligned} &y=0\ :u=\alpha\left(1+\varepsilon e^{i\omega t}\right),\ \ v=-\left(1+\varepsilon\cos\pi z\right),\ w=0,\ \theta=1+\varepsilon e^{i\omega t}, C=1,\\ &y\to\infty:\ u\to 1,\ v\to -1,\ w\to 0,\ p\to p_\infty,\theta\to 0,\quad C\to 0. \end{aligned} \qquad \ldots(17)$$

In the neighbourhood of the plates $u, v, w, p, \theta\, and\, C$ are assumed as given below

$$F(y,z,t)=f_0(y)+\varepsilon f_1(y,z,t)+O\left(\varepsilon^2\right) \qquad \ldots(18)$$

where $F$ stands for any of $u, v, w, p, \theta\, and\, C$.

when $\varepsilon=0$, the problem reduces to the two- dimensional flow with constant suction at the plate. In this case, the equation (11) to (16) are reduced to

$$v_0'=0, \qquad \ldots(19)$$

$$u_0''-v_0u_0'=-Gr\theta_0-GmC_0, \qquad \ldots(20)$$

$$v_0'' - v_0 v_0' = p_0' \qquad \dots(21)$$

$$w_0'' - v_0 w_0' = 0 \qquad \dots(22)$$

$$\theta_0'' - v_0 \Pr \theta_0' = -Ec \Pr \left(u_0'\right)^2 - S\theta_0, \qquad \dots(23)$$

$$C_0'' - v_0 Sc C_0' = 0, \qquad \dots(24)$$

where prime denotes differentiation with respect to y.

The corresponding boundary conditions are

$$\begin{aligned} y = 0 \quad &: \; u_0 = \alpha, \; v_0 = -1, \; w_0 = 0, \; \theta_0 = 1, C_0 = 1, \\ \mathrm{y} \to \infty \; &: \; u_0 = 1, \; v_0 = -1 \; w_0 = 0, \; p_0 = p_{\infty,} \; \theta_0 = 0, \; C_0 = 0. \end{aligned} \qquad \dots(25)$$

Equations (19), (21) and (22) under the boundary conditions (25) give

$$v_0 = -1, \; w_0 = 0 \; and \; p_0 = p_\infty . \qquad ...(26)$$

Equations (20) and (23) are still coupled whose solution can not be determined. Since $Ec$ is small for incompressible fluid flows, therefore $u_0(y)$ *and* $\theta_0(y)$ can be expressed in the powers of $Ec$ as given below

$$\begin{aligned} u_0(y) &= u_{00}(y) + Ecu_{01}(y) + O\left(Ec^2\right), \\ \theta_0(y) &= \theta_{00}(y) + Ec\theta_{01}(y) + O\left(Ec^2\right). \end{aligned} \qquad \dots(27)$$

Using equation (27) into the equations (20) and (23), and equating like powers of $Ec$, we get

$$u_{00}'' - v_0 u_{00}' = -Gr\theta_{00} - GmC_0, \qquad \dots(28)$$

$$u_{01}'' - v_0 u_{01}' = -Gr\theta_{01}, \qquad \dots(29)$$

$$\theta_{00}'' - v_0 \Pr \theta_{00}' + S\theta_{00} = 0, \qquad \dots(30)$$

$$\theta_{01}'' - v_0 \Pr \theta_{01}' + S\theta_{01} = -\Pr\left(u_{00}'\right)^2 . \qquad \dots(31)$$

Now, the corresponding boundary conditions are reduced to

$$\begin{aligned} y = 0 \quad &: \; u_{00} = \alpha, \; u_{01} = 0, \; \theta_{00} = 1, \; \theta_{01} = 0, \\ \mathrm{y} \to \infty \; &: \; u_{00} = 1, \; u_{01} = 0, \; \theta_{00} = 0, \; \theta_{01} = 0. \end{aligned} \qquad \dots(32)$$

Through straight forward calculations, equations (28) to (31) under the boundary conditions (32) are solved and the expressions of $u_0(y), \theta_0(y) and\, C_0(y)$ are given below

$$u_0(y) = \left[1 + (L_1 - L_2)e^{-y} - (1 - \alpha - L_1)e^{-F_1 y} - L_2 e^{-Scy}\right] + Ec\left[L_{10}e^{-y} + L_{11}e^{-F_1 y} + L_{12}e^{-2y} + L_{13}e^{-2F_1 y} + L_{14}e^{-2Scy} + L_{15}e^{-(1+F_1)y} L_{16}e^{-(1+Sc)y} + L_{17}e^{-(F_1+Sc)y}\right], \qquad \ldots(33)$$

$$\theta_0(y) = e^{-F_1 y} + Ec\left[L_3 e^{-F_1 y} + L_4 e^{-2y} + L_5 e^{-2F_1 y} + L_6 e^{-2Scy} + L_7 e^{-(1+F_1)y} + L_8 e^{-(1+Sc)y} + L_9 e^{-(F_1+Sc)y}\right], \qquad \ldots(34)$$

$$C_0(y) = e^{-Scy}. \qquad \ldots(35)$$

when $\varepsilon \neq 0,$ after using (18) into the equations (11) to (16) and comparing the coefficients of like power of $\varepsilon$ we get

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \qquad \ldots(36)$$

$$\frac{1}{4}\frac{\partial u_1}{\partial t} + v_1\frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = Gr\theta_1 + GmC_1 + \left[\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right], \qquad \ldots(37)$$

$$\frac{1}{4}\frac{\partial v_1}{\partial t} - \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \left[\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right], \qquad \ldots(38)$$

$$\frac{1}{4}\frac{\partial w_1}{\partial t} - \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \left[\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right], \qquad \ldots(39)$$

$$\Pr\left[\frac{1}{4}\frac{\partial \theta_1}{\partial t} + v_1\frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y}\right] = \left[\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right] + 2Ec.\Pr.\frac{\partial u_0}{\partial y}.\frac{\partial u_1}{\partial y} + S\theta_1, \qquad \ldots(40)$$

$$\frac{1}{4}\frac{\partial C_1}{\partial t} + v_1\frac{\partial C_0}{\partial y} - \frac{\partial C_1}{\partial y} = \frac{1}{Sc}\left[\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}\right]. \qquad \ldots(41)$$

The corresponding boundary conditions are given by

$$\left.\begin{array}{l} y = 0 \;\; : \;\; u_1 = \alpha e^{i\omega t},\; v_1 = -\cos \pi z,\; w_1 = 0,\; \theta_1 = e^{i\omega t},\; C_1 = 0, \\ y \to \infty : \;\; u_1 = 0,\; v_1 = 0,\; w_1 = 0,\; p_1 = 0,\; \theta_1 =,\; C_1 = 0. \end{array}\right] \qquad \ldots(42)$$

In view of the boundary condition (42), assuming the following

$$u_1(y,z,t) = u_{11}(y)e^{i\omega t} + u_{12}(y)\cos \pi z,$$
$$v_1(y,z,t) = v_{11}(y)e^{i\omega t} + v_{12}(y)\cos \pi z,$$

$$w_1(y,z,t) = -\left[ z v_{11}'(y) e^{i\omega t} + \frac{1}{\pi} v_{12}'(y) \sin \pi z \right],$$

$$p_1(y,z,t) = p_{11}(y) e^{i\omega t} + p_{12}(y) \cos \pi z,$$

$$\theta_1(y,z,t) = \theta_{11}(y) e^{i\omega t} + \theta_{12}(y) \cos \pi z,$$

$$C_1(y,z,t) = C_{11}(y) e^{i\omega t} + C_{12}(y) \cos \pi z. \quad ...(43)$$

It is observed that the equation of continuity (36) is satisfied identically. Substituting equations (43) into the equations (37) to (41) and equating the coefficients of harmonic and non-harmonic terms, we get

$$u_{11}'' + u_{11}' - \frac{i\omega}{4} u_{11} = -Gr\theta_{11} - GmC_{11} + v_{11} u_0', \quad \ldots(44)$$

$$u_{12}'' + u_{12}' - \pi^2 u_{12} = -Gr\theta_{12} - GmC_{12} + v_{12} u_0', \quad \ldots(45)$$

$$v_{11}'' + v_{11}' - \frac{i\omega}{4} v_{11} = p_{11}', \quad \ldots(46)$$

$$v_{11}''' + v_{11}'' - \frac{i\omega}{4} v_{11}' = 0, \quad \ldots(47)$$

$$v_{12}'' + v_{12}' - \pi^2 v_{12} = p_{12}', \quad \ldots(48)$$

$$v_{12}''' + v_{12}'' - \pi^2 v_{12}' = \pi^2 p_{12}, \quad \ldots(49)$$

$$\theta_{11}'' + \Pr \theta_{11}' - \left( \frac{\Pr i\omega}{4} - S \right) \theta_{11} = \Pr v_{11} \theta_0' - 2\,\mathrm{Ec}\Pr u_0' u_{11}', \quad \ldots(50)$$

$$\theta_{12}'' + \Pr \theta_{12}' - \left( \pi^2 - S \right) \theta_{12} = \Pr v_{12} \theta_0' - 2\,\mathrm{Ec}\Pr u_0' u_{12}', \quad \ldots(51)$$

$$C_{11}'' + ScC_{11}' - \frac{i\omega Sc}{4} C_{11} = v_{11}\, \mathrm{Sc} \mathrm{C}_0', \quad \ldots(52)$$

$$C_{12}'' + ScC_{12}' - \pi^2 C_{12} = Scv_{12}\, \mathrm{C}_0'. \quad \ldots(53)$$

The corresponding boundary conditions are reduced to

$$y = 0 \quad : \quad u_{11} = \alpha,\ u_{12} = 0,\ v_{11} = 0,\ v_{12} = -1, v_{11}' = 0,\ v_{12}' = 0,\ \theta_{11} = 1,$$
$$\theta_{12} = 0,\ C_{11} = 0,\ \mathrm{C}_{12} = 0;$$
$$y \to \infty : \ u_{11} = 0,\ u_{12} = 0,\ v_{11} = 0,\ v_{12} = 0,\ p_{11} = 0,\ p_{12} = 0,$$
$$\theta_{11} = 0,\ \theta_{12} = 0,\ C_{11} = 0,\ C_{12} = 0. \quad \ldots(54)$$

Equations (44), (45), (50) and (51) are still coupled second order differential equations. Since Eckert number is $Ec$ small for incompressible fluid flows, therefore $u_{11}, u_{12}, \theta_{11}$ *and* $\theta_{12}$ can be expressed in the powers of $Ec$ as given below

$$\left.\begin{aligned} u_{11}(y) &= u_{110}(y) + Ecu_{111}(y) + O\left(Ec^2\right), \\ u_{12}(y) &= u_{120}(y) + Ecu_{121}(y) + O\left(Ec^2\right), \\ \theta_{11}(y) &= \theta_{110}(y) + Ec\theta_{111}(y) + O\left(Ec^2\right), \\ \theta_{12}(y) &= \theta_{120}(y) + Ec\theta_{121}(y) + O\left(Ec^2\right). \end{aligned}\right] \quad \ldots(55)$$

Substituting (55) into the equations (44), (45), (50) and (51) and equating the coefficient of like powers of $Ec$ and neglecting the terms in $O\left(Ec^2\right)$ and higher order terms , we get

$$u''_{110} + u'_{110} - \frac{i\omega}{4} u_{110} = -Gr\theta_{110} - GmC_{11} + v_{11}u'_{00}, \quad \ldots(56)$$

$$u''_{111} + u'_{111} - \frac{i\omega}{4} u_{111} = -Gr\theta_{111} + v_{11}\,\mathrm{u}'_{01}, \quad \ldots(57)$$

$$u''_{120} + u'_{120} - \pi^2 u_{120} = -Gr\theta_{120} - GmC_{12} + v_{12}u'_{00}, \quad \ldots(58)$$

$$u''_{121} + u'_{121} - \pi^2 u_{121} = -Gr\theta_{121} + v_{12}u'_{01}, \quad \ldots(59)$$

$$\theta''_{110} + \Pr\theta'_{110} - \left(\frac{\Pr i\omega}{4} - S\right)\theta_{110} = \Pr v_{11}\theta'_{00}, \quad \ldots(60)$$

$$\theta''_{111} + \Pr\theta'_{111} - \left(\frac{\Pr i\omega}{4} - S\right)\theta_{111} = \Pr v_{11}\theta'_{01} - 2\Pr u'_{00}u'_{110}, \quad \ldots(61)$$

$$\theta''_{120} + \Pr\theta'_{120} - \left(\pi^2 - S\right)\theta_{120} = \Pr v_{12}\theta'_{00}, \quad \ldots(62)$$

$$\theta''_{121} + \Pr\theta'_{121} - \left(\pi^2 - S\right)\theta_{121} = \Pr v_{12}\theta'_{01} - 2\Pr u'_{00}u'_{120}. \quad \ldots(63)$$

Now, the corresponding boundary conditions are reduced to

$$\left.\begin{aligned} y = 0 \quad &: \; u_{110} = \alpha,\; u_{111} = 0,\; u_{120} = 0,\; u_{121} = 0,\; \theta_{110} = 1,\; \theta_{111} = 0, \\ &\quad \theta_{120} = 0,\; \theta_{121} = 0; \\ y \to \infty &: \; u_{110} = 0,\; u_{111} = 0,\; u_{120} = 0,\; u_{121} = 0,\; \theta_{110} = 0,\; \theta_{111} = 0, \\ &\quad \theta_{120} = 0,\; \theta_{121} = 0. \end{aligned}\right] \quad \ldots(64)$$

Through straight forward calculations, the solutions of $u_1(y), v_1(y), w_1(y), p_1(y),\ \theta_1(y)$ *and* $C_1(y)$ are obtained and given by

$$u_1(y,z,t) = [(\alpha - L_{21})e^{-F_4 y} + L_{21}e^{-F_3 y} + Ec\{L_{29}e^{-F_4 y} + L_{30}e^{-F_3 y} + L_{31}e^{-(1+F_4)Y} + L_{32}e^{-(1+F_3)y} + L_{33}e^{-(F_1+F_4)y} + L_{34}e^{-(F_1+F_3)y} + L_{35}e^{-(F_4+Sc)y} + L_{36}e^{-(F_3+Sc)y}\}]e^{i\omega t} + [\{L_{40}e^{-ny} + L_{41}e^{-F_5 y} + L_{42}e^{-(n+F_1)y} + L_{43}e^{-(\pi+F_1)y} + L_{44}e^{-F_2 y} + L_{45}e^{-(n+Sc)y} + L_{46}e^{-(\pi+Sc)y} + L_{47}e^{-(n+1)y} + L_{48}e^{-(\pi+1)y}\} + Ec\{L_{72}e^{-ny} + L_{73}e^{-(n+1)y} + L_{74}e^{-(\pi+1)y} + L_{75}e^{-(n+F_1)y} + L_{76}e^{-(\pi+F_1)y} + L_{77}e^{-(n+2)y} + L_{78}e^{-(\pi+2)y} + L_{79}e^{-(n+2F_1)y} + L_{80}e^{-(\pi+2F_1)y} + L_{81}e^{-(n+2Sc)y} + L_{82}e^{-(\pi+2Sc)y} + L_{83}e^{-(1+n+F_1)y} + L_{84}e^{-(1+\pi+F_1)y} + L_{85}e^{-(1+n+Sc)y} + L_{86}e^{-(1+\pi+Sc)y} + L_{87}e^{-(n+F_1+Sc)y} + L_{88}e^{-(\pi+F_1+Sc)y} + L_{89}e^{-(n+Sc)y} + L_{90}e^{-(1+F_5)y} + L_{91}e^{-(F_1+F_5)y} + L_{92}e^{-(F_5+Sc)y} + L_{93}e^{-(1+F_2)y} + L_{94}e^{-(F_1+F_2)y} + L_{95}e^{-(F_2+Sc)y}\}]\cos\pi z, \quad ...(65)$$

$$v_1(y,z,t) = \frac{1}{(n-\pi)}\left[\pi e^{-ny} - ne^{-\pi y}\right]\cos\pi z, \quad ...(66)$$

$$w_1(y,z,t) = \frac{n}{(n-\pi)}\left[e^{-ny} - e^{-\pi y}\right]\sin\pi z, \quad ...(67)$$

$$p_1(y,z,t) = \frac{n}{(\pi-n)}e^{-\pi y}\cos\pi z, \quad ...(68)$$

$$\theta_1(y,z,t) = [e^{-F_3 y} + Ec\{L_{22}e^{-F_3 y} + L_{23}e^{-(1+F_4)Y} + L_{24}e^{-(1+F_3)y} + L_{25}e^{-(F_1+F_4)y} + L_{26}e^{-(F_1+F_3)y} + L_{27}e^{-(F_4+Sc)y} + L_{28}e^{-(F_3+Sc)y}\}]e^{i\omega t} + [\{L_{37}e^{-F_5 y} + L_{38}e^{-(n+F_1)y} + L_{39}e^{-(\pi+F_1)y}\} + Ec\{L_{49}e^{-F_5 y} + L_{50}e^{-(n+F_1)y} + L_{51}e^{-(\pi+F_1)y} + L_{52}e^{-(n+2)y} + L_{53}e^{-(\pi+2)y} + L_{54}e^{-(n+2F_1)y} + L_{55}e^{-(\pi+2F_1)y} + L_{56}e^{-(n+2Sc)y} + L_{57}e^{-(\pi+2Sc)y} + L_{58}e^{-(1+n+F_1)y} + L_{59}e^{-(1+\pi+F_1)y} + L_{60}e^{-(1+n+Sc)y} + L_{61}e^{-(1+\pi+Sc)y} + L_{62}e^{-(n+F_1+Sc)y} + L_{63}e^{-(\pi+F_1+Sc)y} + L_{64}e^{-(n+1)y} + L_{65}e^{-(n+Sc)y} + L_{66}e^{-(1+F_5)y} + L_{67}e^{-(F_1+F_5)y} + L_{68}e^{-(F_5+Sc)y} + L_{69}e^{-(1+F_2)y} + L_{70}e^{-(F_1+F_2)y} + L_{71}e^{-(F_2+Sc)y}\}]\cos\pi z, \quad ...(69)$$

$$C_1(y,z,t) = [L_{18}e^{-F_2 y} + L_{19}e^{-(n+Sc)y} + L_{20}e^{-(\pi+Sc)y}]\cos\pi z. \quad ...(70)$$

Substituting the expressions of $u_0, u_1, \theta_0, \theta_1, C_0$ *and* $C_1$ into the equation (18), the final form of velocity, temperature and concentration distributions are obtained.

## SKIN FRICTION COEFFICIENT

Skin friction coefficient at the plate along $x^*$ *and* $z^*$ - directions are given by

In $x^*$ - direction

$$C_{f_x} = \left.\frac{\tau_{x^* y^*}}{\rho U_\infty V_0}\right|_{y^*=0} = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$= \left(L_{96} + EcL_{97}\right) + \varepsilon\left[\left(L_{98} + EcL_{99}\right)e^{i\omega t} + \left(L_{100} + EcL_{101}\right)\cos\pi z\right], \qquad \ldots(71)$$

where $\tau_{x^*y^*} = \mu\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)_{y^*=0}$ is the shear stress at the plate.

In $z^*$ - direction

$$C_{f_z} = \left.\frac{\tau_{y^*z^*}}{\rho U_\infty V_0}\right|_{y^*=0} = \bar{\lambda}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)_{y=0},$$

$$= \varepsilon\bar{\lambda}\left(\pi - n\right)\sin\pi z, \qquad \ldots(72)$$

where $\tau_{y^*z^*} = \mu\left(\frac{\partial w^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*}\right)_{y^*=0}$ is the shear stress at the plate

## NUSSELT NUMBER

The rate heat transfer in terms of Nusselt number at plate is given by

$$Nu = \frac{q\upsilon}{\kappa V_0\left(T_w - T_\infty\right)} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0},$$

$$= \left(F_1 + EcL_{102}\right) + \varepsilon\left[\left(F_3 + EcL_{103}\right)e^{i\omega t} + \left(L_{104} + EcL_{105}\right)\cos\pi z\right], \qquad \ldots(73)$$

where $q = -\kappa\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0}$ is the rate of heat transfer at the plate.

## SHERWOOD NUMBER

The rate of mass transfer coefficient in terms of Sherwood number is given by

$$Sh = \frac{m\upsilon}{\left(C_w - C_\infty\right)DV_0} = \left(\frac{\partial C}{\partial y}\right)_{y=0},$$

$$= Sc + \varepsilon L_{106}\cos\pi z, \qquad \ldots(74)$$

where $m = -D\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}$ is the rate of mass transfer at the plate.

The expressions of constants occurred in $u, \theta, C, C_{f_x}, C_{f_z}, Nu\, and\, Sh$ are not given here for sake of brevity.

## RESULTS AND DISCUSSION

In order to get physical insight into the problem, the numerical calculations for velocity distribution, temperature distribution, species concentration distribution, skin-friction coefficient at the plate, rate of heat transfer in terms of Nusselt number at the plate and rate of mass transfer in terms of Sherwood number at the plate are obtained for different values of the physical parameters and demonstrated in graphs and tables.

It is observed from figure 1 that fluid velocity increases due to increase in Grashof number, modified Grashof number or heat source parameter. Figure 2 illustrates that fluid velocity increases due to increase in Eckert number while it decreases with the increase in Prandtl number or Schmidt number. It is seen from figure 3 that fluid temperature decreases due to increase in Prandtl number or Schmidt number whereas it increases due to increase in Eckert number. Figure 4 represents the fluid temperature increase due to increase in Grashof number, modified Grashof number or heat source parameter. It is noted from figure 5 that concentration profiles decreases due to increase in Schmidt number.

It is observed from table 1 that skin-friction coefficient at the plate along $x^*$ - direction increases due to increase in the Grashof number, modified Grashof number, Eckert number or heat source parameter, while it decreases due to increase in Prandtl number or Schmidt number. Nusselt number at the plate increases due to increase in Schmidt number, while it decreases due to increase in Grashof number, modified Grashof number, Prandtl number, Eckert number or heat source parameter. The Sherwood number at the plate increases due to increase in the Schmidt number.

## CONCLUSIONS

In view of graphs and tables the following conclusion are made

1. Thermal and concentration buoyancy effects accelerates the velocity of fluid particle.
2. Fluid velocity increases due to increase in heat source parameter or Eckert number.
3. Increase in Prandtl number or Schmidt number results in decrease in fluid velocity.
4. Eckert number increases fluid temperature while Prandtl number or Schmidt number decrease fluid temperature.
5. An increase in heat source parameter leads to increase in fluid temperature.
6. Concentration profiles decreases due to increase in Schmidt number.

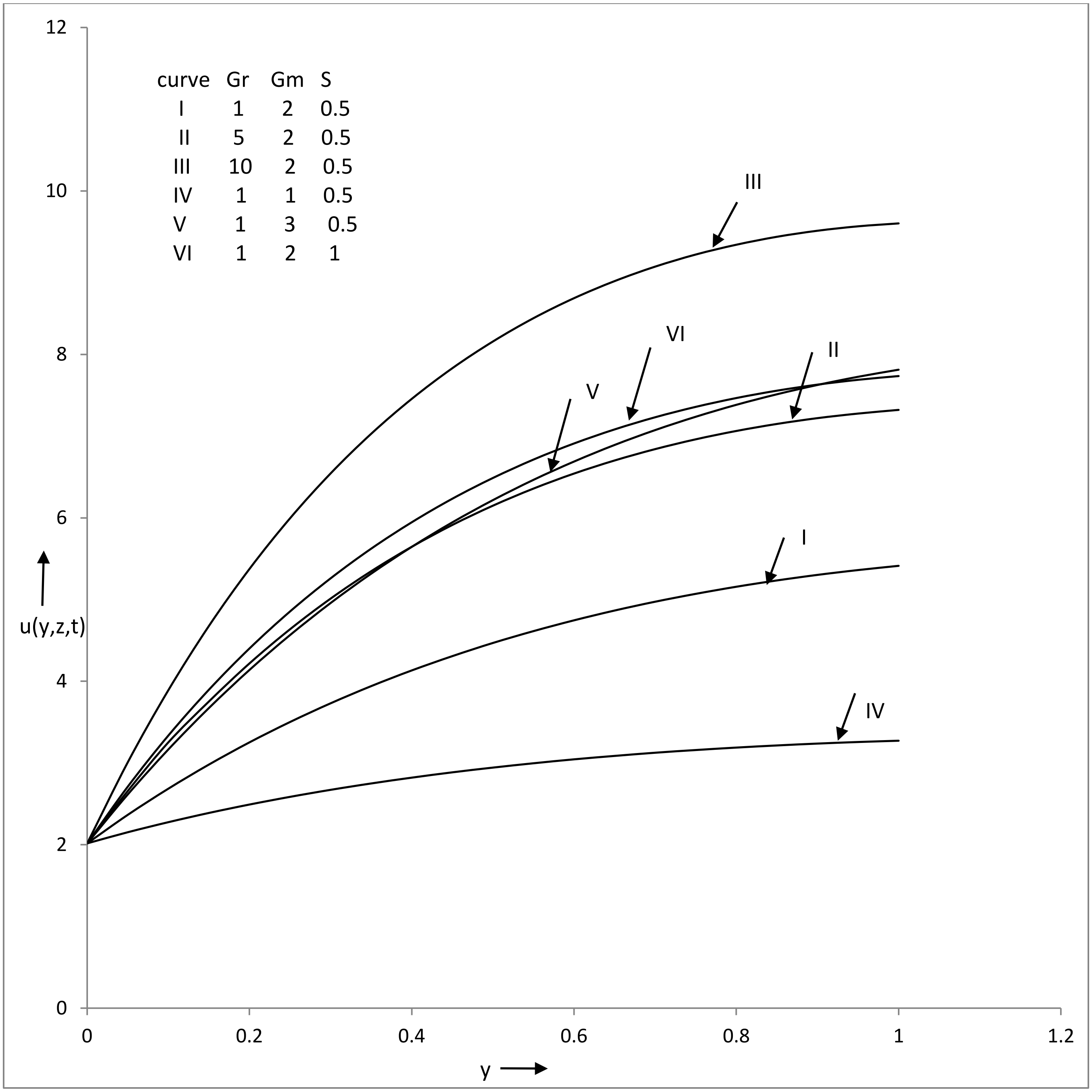


**Figure 1** Velocity profiles for different values of $Gr, Gm\ and\ S$ when $\Pr = 5,\ Ec = 0.01,$ $Sc = 0.3,\ \omega = 5,\ \varepsilon = 0.01,\ \mathrm{z} = 1/6,\ \alpha = 2\ and\ \omega t = \pi / 6.$

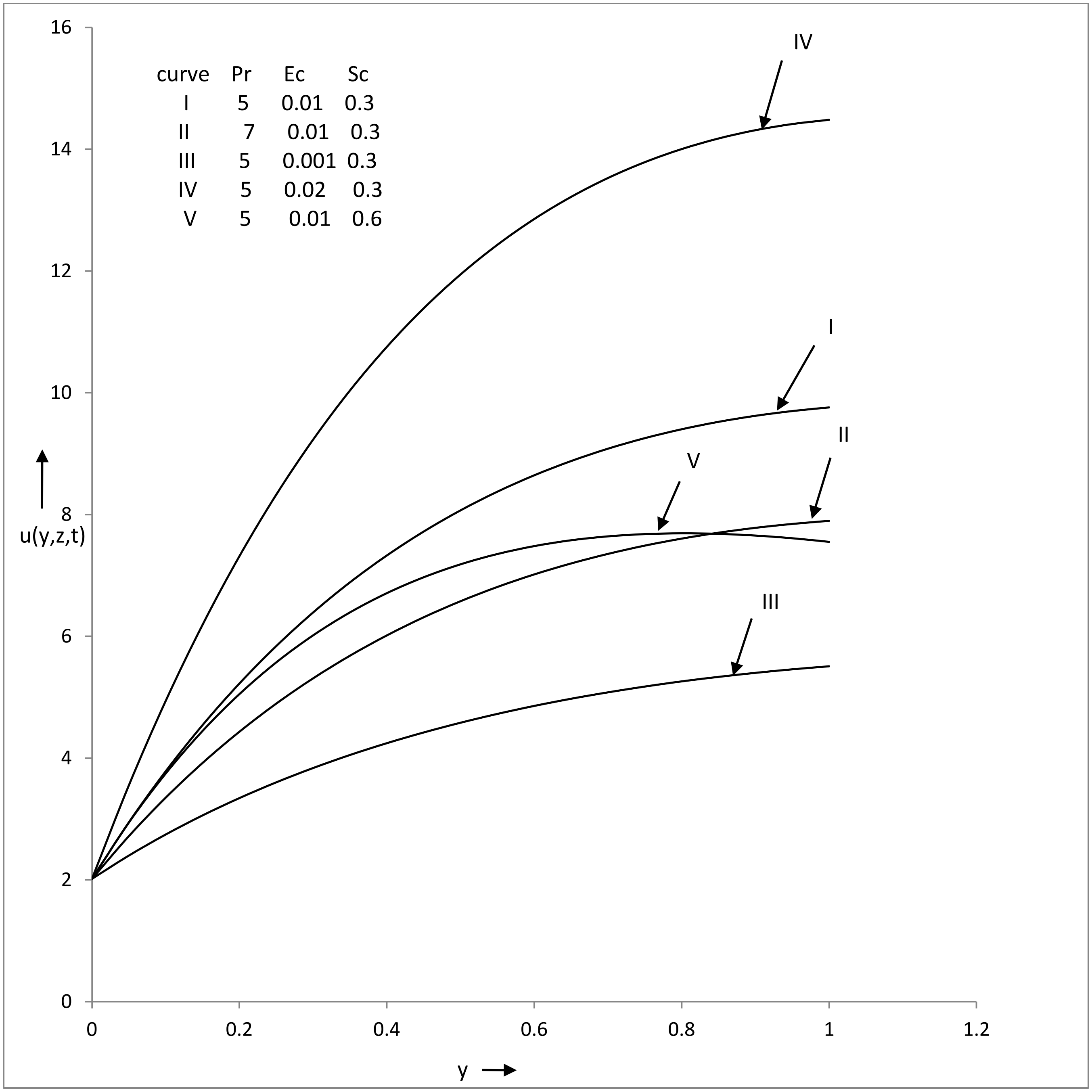


**Figure 2** Velocity profiles for different values of $Pr, Ec$ *and* $Sc$ when $\mathrm{Gr}=5,\ Gm=2,$
$S=2,\ \omega=5,\ \varepsilon=0.01,\ \mathrm{z}=1/6,\ \alpha=2$ *and* $\omega t=\pi/6.$

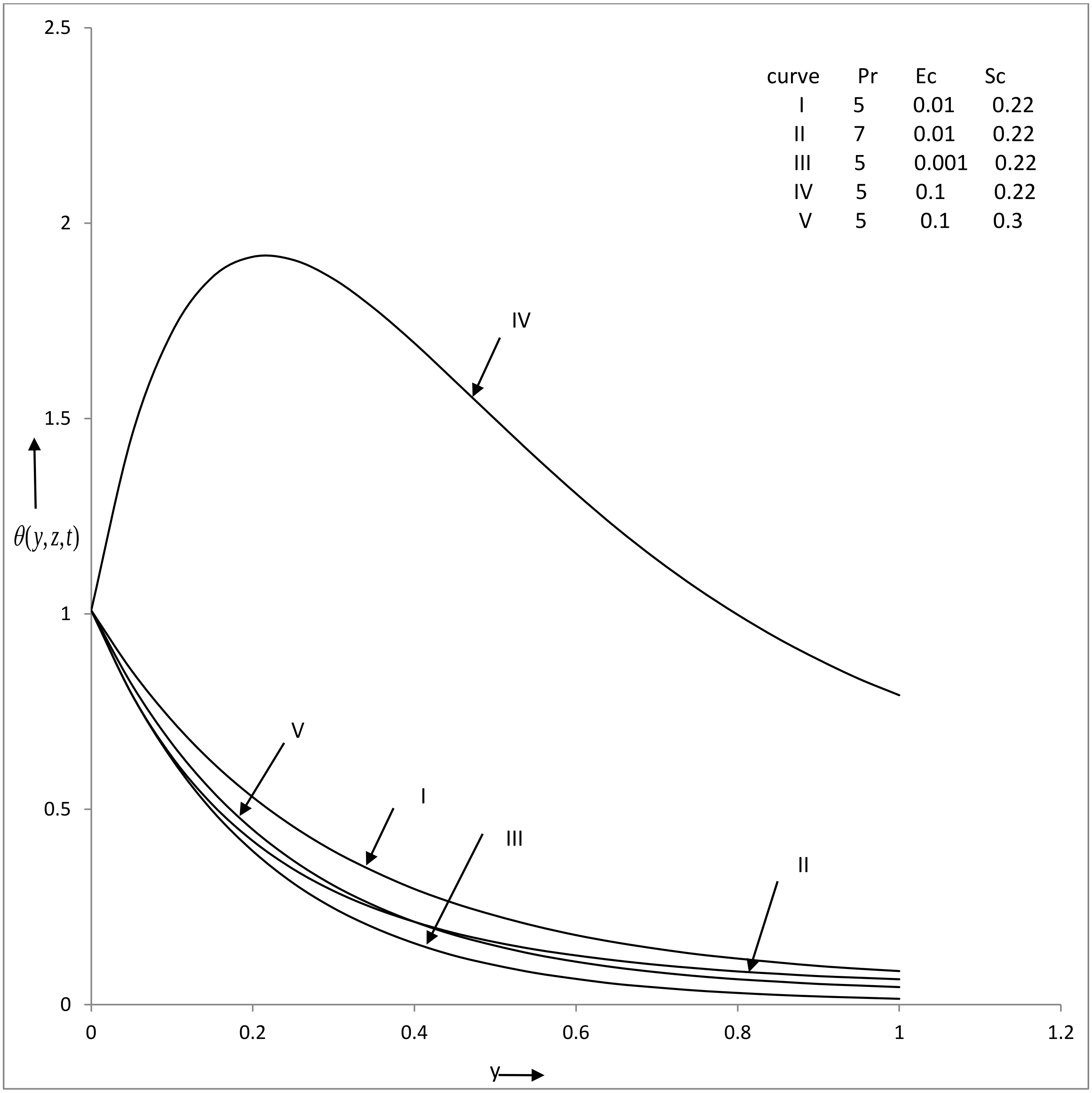


**Figure 3** Temperature profiles for different values of $Pr, Ec$ *and* $Sc$ when $Gr = 5,\ Gm = 2,\ S = 0.5,\ \omega = 5,\ \varepsilon = 0.01,\ z = 1/6,\ \alpha = 2$ *and* $\omega t = \pi/6$.

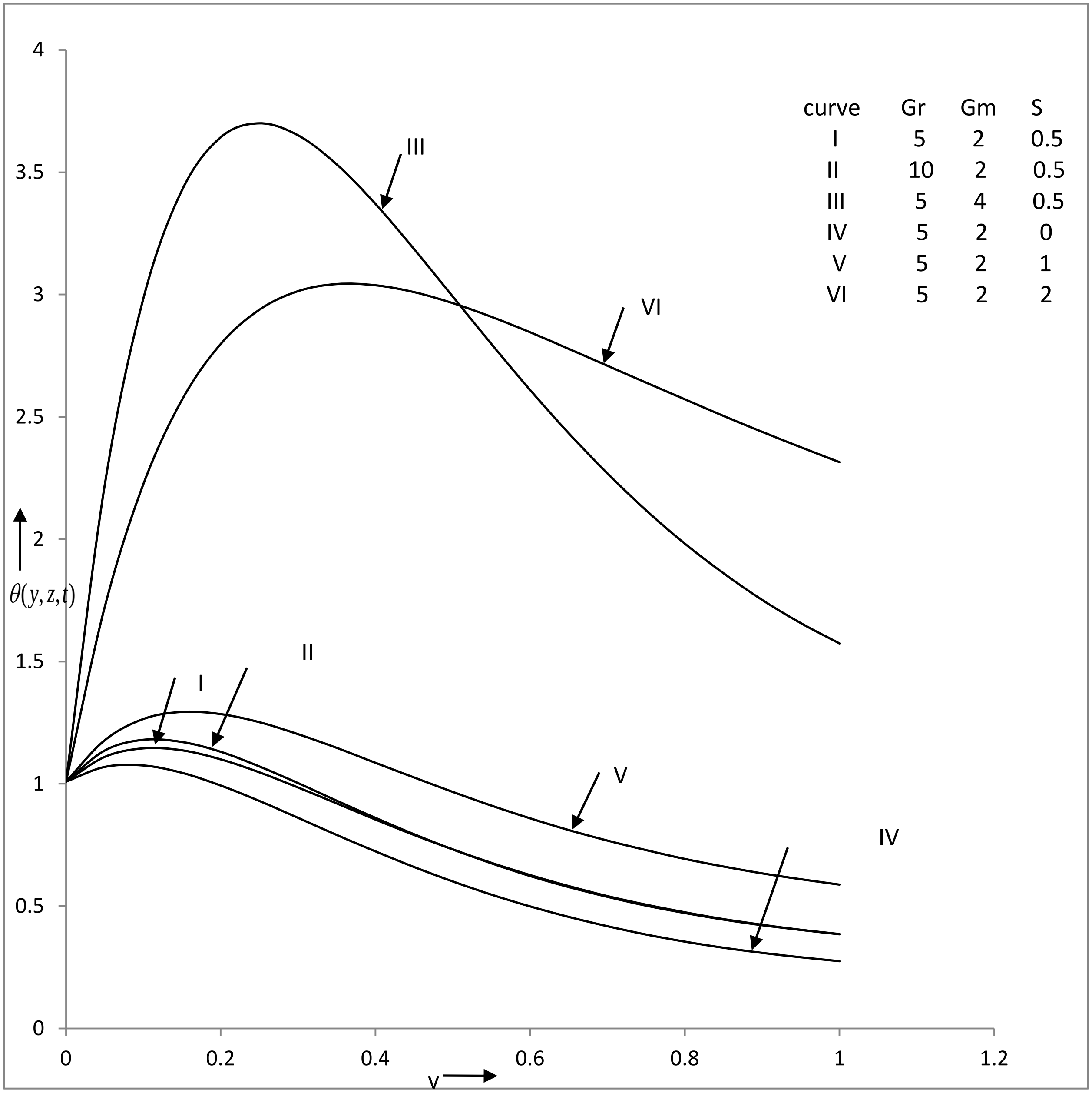


**Figure 4** Temperature profiles for different values of $Gr, Gm$ *and* $S$ when $\Pr = 5,\ Ec = 0.1,$ $Sc = 0.3,\ \omega = 5,\ \varepsilon = 0.01,\ z = 1/6,\ \alpha = 2$ *and* $\omega t = \pi/6.$

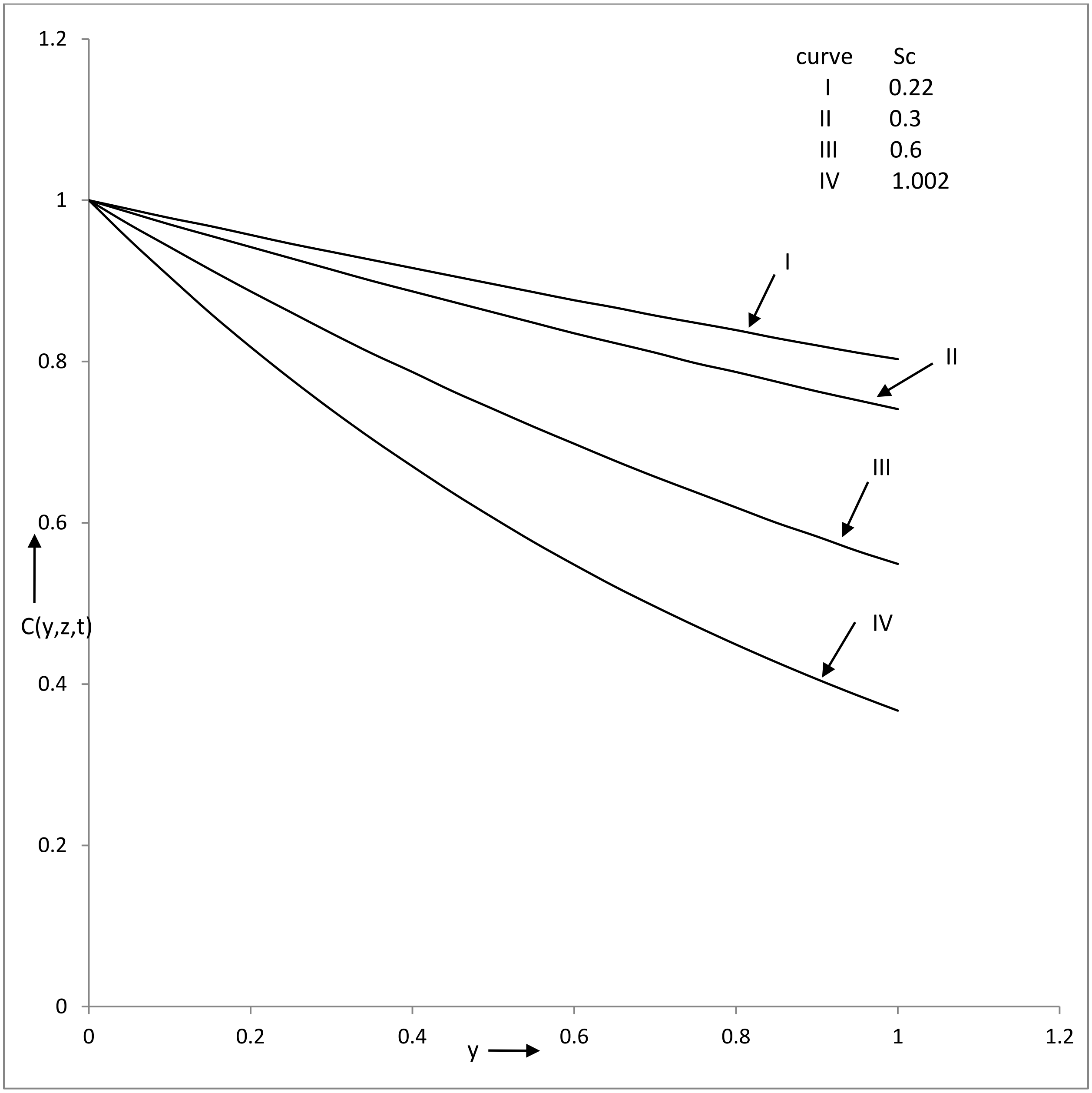


**Figure 5** Concentration profiles for different values of $Sc, when\ \varepsilon = 0.01\ and\ z = 1/6.$

**Table 1.** Numerical values of skin-friction coefficient, Nusselt number and Sherwood number at the plate for various values of physical parameters

| $Gr$ | $Gc$ | Pr | $Ec$ | $S$ | $Sc$ | $\overline{\lambda}$ | $\varepsilon$ | $C_{f_x}$ | $C_{f_z}$ | $Nu$ | $Sh$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 2 | 5 | 0.01 | 0.5 | 0.22 | 1 | 0.01 | 10.858 | -0.0027 | -1.539 | 0.22 |
| 10 | 2 | 5 | 0.01 | 0.5 | 0.22 | 1 | 0.01 | 13.649 | -0.0027 | -1.636 | 0.22 |
| 5 | 4 | 5 | 0.01 | 0.5 | 0.22 | 1 | 0.01 | 25.38 | -0.0027 | -6.895 | 0.22 |
| 5 | 2 | 7 | 0.01 | 0.5 | 0.22 | 1 | 0.01 | 10.374 | -0.0027 | -2.001 | 0.22 |
| 5 | 2 | 5 | 0.1 | 0.5 | 0.22 | 1 | 0.01 | 26.557 | -0.0027 | -16.005 | 0.22 |
| 5 | 2 | 5 | 0.01 | 1 | 0.22 | 1 | 0.01 | 11.86 | -0.0027 | -1.915 | 0.22 |
| 5 | 2 | 5 | 0.01 | 0.5 | 0.6 | 1 | 0.01 | 3.49 | -0.0027 | -0.072 | 0.601 |
| 5 | 2 | 5 | 0.01 | 0.5 | 0.22 | 2 | 0.01 | 10.858 | -0.0054 | -1.539 | 0.22 |
| 5 | 2 | 5 | 0.01 | 0.5 | 0.22 | 1 | 0.04 | 10.869 | -0.011 | -1.334 | 0.22 |

## REFERENCES


1. **Bansal, J.L.** ' Viscous fluid dynamics'. Oxford and IBH Pub. Co., New Delhi, 1977.

2. **Champkha, A.J.** 'MHD of a uniformly stretched vertical permeable surface in the presence of heat generation / absorption and a chemical reaction'. Int. Comm. Heat Mass Transfer Vol. 30, 2003, pp. 413-422.

3. **Fayza Mohammed Nasser El-Fayez** 'Effects of chemical reaction on the unsteady free convection flow past an infinite permeable moving plate with variable temperature '. JSEMAT, Vol. 2,2012, pp.100-109.

4. **Gersten, K. and Gross,J.F.** 'Flow and heat transfer along a plane wall with periodic suction '. ZAMP, Vol. 25, 1974, pp.339.

5. **Ibrahim, F.S.,Elaiw, A.M. and Bak, A.A.R.** ' Effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction'. Comm. In non linear Sc. Num. Simulation, Vol. 13, 2008, pp. 1056-1066.

6. **Kim, Youn,J.** 'Unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate with variable suction'. Int. J. Eng., Sci. Vol. 28, 2000, pp. 833-845.

7. **Lian-Cun-Zheng, Xin Jin, Xin-Xin Zhange and Jun Hong Zhang** ' Unsteady heat and mass transfer in MHD flow over an oscillatory stretching surface with Soret and Dufour effects'. Acta Mechanica Sinica, Vol. 29, 2013, pp. 667-675.

8. **Mankinde, O.D.** 'Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate'. International Communications in Heat and Mass Transfer, Vol. 32, 2005, pp. 1411-1419

9. **Mankinde, O.D. and Ogulu, A.** ' The effect of thermal radiation on heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field'. Chem. Engg. Comm., Vol. 195, 2008, pp.1575-1584.

10. **Pai, S.I.** 'Viscous flow theory – I : Laminar flow'. D. Van Nostrand Comp. Inc., New York, 1956.

11. **Patil, P.M. and Kulkarni, P.S** ' Effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat

generation'. Int. J. Thern. Sci., Vol. 47, 2008, pp. 1043-1054.

12. **Raptis,A., and Massals, C.V.** ' Magneto hydrodynamics flow past a plate by the presence of radiation '. Heat and Mass Transfer, Vol. 34, 1998, pp. 107-109.

13. **Schlichting, H. and Gersten, K.** ' Boundary Layer Theory'. Springer Verlag, Berlin, 2000.

14. **Sharma, P.R., Gaur, M. and Gaur, Y.N.** 'Three- dimensional viscous flow and heat transfer along a porous plate in the presence of sinusoidal suction and temperature'. Appl. Science Periodical, India, Vol. 4, 2002, pp. 141-146.

15. **Sharma, P.R. and Katta, Rachana** 'Mass transfer effect on unsteady mixed convective flow and heat transfer along an infinite vertical plate bounded with porous medium'. J. Ultra Scientist, India, Vol. 23(1) M, 2011, pp.75-90.

16. **Singh. K.D.** ' Three-dimensional viscous flow and heat transfer along a porous plate'. ZAMM, VOl. 73, 1993, pp.58.

17. **Singh, P. Sharma, V.P and Mishra, U.N.** ' Three-dimensional fluctuating flow and heat transfer along a plate with suction'. Int. J. Heat Mass Transfer, Vol. 21, 1978, pp. 1117.