MONOTONE REPAIR REPLACEMENT POLICY FOR A COLD STANDBY REPAIRABLE SYSTEM EXPOSING TO EXPONENTIAL FAILURE LAW

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Abstract

This paper studies a cold standby repairable system consisting of two identical components namely component 1, component 2 and one repairman is studied. Assume that each component after repair is not 'as good as new' and also the successive working times form a decreasing α -series process, the successive repair time's form an increasing geometric process and both the processes are exposing to exponential failure law. Under these assumptions we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. It can be determined that an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. It can also be derived an explicit expression of the long-run average cost and the corresponding optimal replacement policy N* can be determined analytically. Numerical results are provided to support the theoretical results.

Key words: *Monotone processes, Replacement policy N, Alpha series process, Geometric process, Renewal theorem stochastic process.*

1. Introduction

Many replacement models were developed under the assumption that the system after repair is "as good as new".This leads to a perfect repair model. But it is not always true for deteriorating systems due to ageing and accumulated wear. Later, Barlow and Hunter [1] developed a minimal repair model in which the minimal repair does not change the age of the system.

Brown and Proschan [2] proposed an imperfect repair model under which the repair will be perfect repair with probability 'p' and with probability '(1-p)' as a minimal repair. Much research work has been carried out by Block et al [4] and others have also worked in this direction. It is reasonable to assume that the successive working times of the deteriorating systems after repair will become shorter and shorter, while the consecutive repair time of the system will become longer and longer. Finally it can't work any longer, neither can it be repaired. To model such a deteriorating repairable system Lam [5] first introduced a geometric process repair model in which he studied two kinds of replacement policies, one based on the working age T of the system and the other based on the failure number N of the system. He derived an explicit expression for the long run average cost per unit time under these two kinds of policies and also proved optimal policy N* is better than the optimal policy T*. Stadje and Zuckerman [7] presented a general monotone process to generalize Lam's work. Latter much research work has been carried out by using geometric process to generalize Lam's work and corresponding optimal replacement policies were developed by Zhang [9,10], Wang and Zhang[8], Zhang and Wang [11], Zhang at al [12-14] determined an optimal replacement policy for a deteriorating production system with preventive maintenance by generalizing Lam's [5] work. Many optimal replacement policies were also developed for cold standby repairable systems using geometric processes.

Zhang [10] considered a cold standby repairable system consisting of two identical components and one repairman. He developed two kinds of repair replacement policies, one based on the working age T of component 1 under which the system is replaced when working age of component 1 reaches T and the other based on failure number N of component 1 under which the system is replaced when the failure number of component 1 reaches N. He derived an explicit expression for long-run average cost per unit time of the system under these two kinds of policies.

However the geometric process is more useful model for deteriorating system, Braun et al [2005] introduced an alternative model, the α -series process ,which contributes these characteristics. Furthermore Braun et al [3] explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The α -series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the α -series process does. Braun et al [3] also presented that both the increasing geometric process and the α -series process have a finite first moment under certain general conditions. However the decreasing geometric process usually has an infinite first moment under certain conditions. Thus the decreasing α -series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

Based on this understanding this paper studies a cold standby repairable system consisting of two identical components namely component 1, component 2 and one repairman is studied. Assume that each component after repair is not 'as good as new' and also the successive working times form a decreasing α -series process, the successive repair time's form an increasing geometric process and both the processes are exposing to exponential failure law. Under these assumptions we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. It can be determined that an optimal repair replacement policy N* such that the long run average cost per unit time is minimized. It can also be derived an explicit expression of the long-run average cost and the corresponding optimal replacement policy N* can be determined analytically. Numerical results are provided to support the theoretical results.

In modeling these deteriorating systems, the definitions according to Lam [5], are considered.

2. The Model

In this section, an optimal replacement policy N for a cold standby repairable system using geometric process exposing to exponential failure law is studied under the following assumptions:

Assumptions:

1. At the beginning two components are good. The component 1 works while component 2 is under cold standby.
2. As soon as the working component fails, it is immediately repaired by the repairman. At the same time, standby one begins to work. When the failed one has been repaired, it either begins to work again or becomes cold standby. If one fails another is still under repair, it must wait for repair and the system breaks down.
3. The replacement time is negligible.
4. Each component after repair is not 'as good as new'.

5. The time interval between the completion of the (n-1)th repair and the completion of the nth repair on component 'i' is called nth cycle of component i, for i=1, 2 and n=1,2,…
6. Let $X_n^{(i)}$ and $Y_n^{(i)}$ are all independent, for i=1, 2 and n=1, 2, 3….. .
7. Let $X_n^{(i)}$ and $Y_n^{(i)}$ be successive working time follows decreasing a α -series process, the successive repair times form an increasing geometric process respectively and both the processes are exposing to exponential failure law. Where i=1, 2 and n=1, 2, 3….. .
8. Let F(k^{α} x) and G (b^{k-1} y) be the distribution function of $X_n^{(i)}$ and $Y_n^{(i)}$ respectively, for i=1,2 and n=1,2,…. where a > 1 and 0 < b < 1.
9. $E(X_n^{(i)}) = \dfrac{\lambda}{k^{\alpha}}$ *and* $E(Y_n^{(i)}) = \dfrac{\mu}{a^{k-1}}$ *for* $i = 1,2.$
10. $E(X_1^{(i)}) = \lambda$ and $\mathrm{E}(\mathrm{Y}_1^{(\mathrm{i})}) = \mu,$ for $i = 1,2.$.
11. The cold standby state and nearest working state have the same distribution. Similarly the waiting time for repair state and repair period have the same distribution.
12. The component in the system can't produce the working reward while in cold standby state, and no cost is incurred during waiting for repair.
13. The repair cost rate of the each component is C_r, the working reward rate of each component is C_w, and the replacement cost of the system is C.

Under these assumptions, an explicit expression for the long-run average cost per unit time and optimal solution for obtaining number of failures (N), which minimizes the long-run average cost per unit time, is discussed below.

3. The Long-run Average Cost Rate Under Policy N

There are two kinds of repair replacement policies: one based on the working age T of component 1 under which we replace the system when the working age of component 1 reaches T and the other based on the failure number (N) of component 1 under which we replace the system when failure number of component 1 reaches N. But here the replacement policy N is considered because it is very effective and easy to implement.

Thus number of failures of component 1 reaches N, then component 2 is either under working state or under waiting for repair state in the Nth cycle. Naturally, the farmer works until failure in the Nth cycle. The latter is not repaired any more in the Nth cycle, while component 1 works in the (N+1)th cycle.

Let T_n ($n \geq 2$) be the time between the (n-1)th replacement and the nth replacement of the system under policy N. Clearly {T_1, T_2…. } form a renewal process and the inter arrival time between two consecutive replacements is called renewal cycle.

According to renewal reward theorem Ross [6], the long-run average cost rate under policy N is:

$$C(N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of the renewal cycle}}. \qquad (3.1)$$

Let L be the length of renewal cycle of the system under policy N, then

$$L=\sum_{k=1}^{N+1}X_K^{(1)}+\sum_{k=1}^{N}Y_K^{(1)}+\sum_{k=2}^{N}\left[Y_{K-1}^{(2)}-X_K^{(1)}I_{(Y_{K-1}^{(2)}-X_K^{(1)}>0)}\right]$$
$$+\sum_{k=1}^{N}\left[X_K^{(2)}-Y_K^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right], \quad (3.2)$$

where the first, second, third and fourth terms respectively working age, repair time, waiting for repair and cold standby time of component 1 and where I is an indicator random variable such that

$$I_A=\begin{cases}1 & \text{if event A occurs.}\\ 0 & \text{if event A does't occurs.}\end{cases}$$

Now we find expected value of renewal cycle length L, under the assumptions of the model.

$$E(L)=E\left[\sum_{k=1}^{N+1}X_k^{(1)}\right]+E\left[\sum_{k=1}^{N}Y_k^{(1)}\right]+E\left[\sum_{k=2}^{N}(Y_{k-1}^{(2)}-X_k^{(1)})I_{\{Y_{k-1}^{(2)}-X_k^{(1)}>0\}}\right]+E\left[\sum_{k=1}^{N}\left(X_k^{(2)}-Y_k^{(1)}\right)I_{\{X_k^{(2)}-Y_k^{(1)}>0\}}\right]. \quad (3.3)$$

Now the expected length of working time can be obtained as follows:

Let $X_k^{(i)} \sim \exp(\lambda)$ $\;for\; k=1,2,3,...., \;and\; i=1,2.$

Then the distribution function of $X_k^{(i)}$, for k=1,2,3,….and i=1,2 is :

$$F_k(x)=F(k^{\alpha}x)=1-e^{-\left(\frac{k^{\alpha}x}{\lambda}\right)};x>0,\lambda>0 \quad (3.4)$$

By definition the expected length of working time is :

$$E\left(X_k^{(i)}\right)=\int_0^{\infty}xdF\left(k^{\alpha}x\right), \qquad i=1,2. \quad (3.5)$$

$$=\frac{\lambda}{k^{\alpha}},\; where\; i=1,2. \quad (3.6)$$

The expected length of repair time of component 1 can be obtained as follows:

Let $Y_k^{(i)} \sim \exp(\mu)$ then the distribution function of $Y_k^{(i)}$ for i=1,2, and k=1, 2, 3, …., is

$$F_k(y)=F(b^{k-1}y)=1-e^{-\left(\frac{b^{k-1}y}{\mu}\right)};y>0,\;\mu>0 \quad (3.7)$$

By definition, the expected length of repair time is:

$$E\left(Y_k^{(i)}\right)=\int_0^{\infty}ydF(b^{k-1}y) \qquad i=1,2.$$

$$= \frac{\mu}{b^{k-1}}, \qquad i = 1,2. \tag{3.8}$$

The expected length of waiting time for repair can be computed as follows:

Let g(u) be the probability density function of $u = Y_{k-1}^{(2)} - X_k^{(1)}$, then by definition of probability density function and using Jacobian transformation we have:

$$g(u) = \int_0^\infty f\left(v, u+v\right) dv,$$

where $X_k^{(1)} = v,\ \mathrm{Y}_{k-1}^{(2)} = u+v,$ such that $u = Y_{k-1}^{(2)} - X_k^{(1)}$. (3.9)

Since $X_k^{(i)}$ *and* $Y_k^{(i)}$ are all independent, for i=1,2 and k=1,2,3,…..,n.

$$g(u) = \int_0^\infty f\left(v\right).f\left(u+v\right) dv. \tag{3.10}$$

From equations (3.9) and (3.10) we have:

$$g(u) = \frac{k^\alpha b^{k-2}\lambda\mu}{k^\alpha\lambda + b^{k-2}\mu} e^{-b^{k-2}\mu u}; \quad for \quad u \geq 0 \cdot \tag{3.11}$$

Let $E\left[Y_{k-1}^{(2)} - X_k^{(1)} I_{\{Y_{K-1}^{(2)} - X_K^{(1)} > 0\}}\right] = \int_0^\infty u g(u) du$, (3.12)

$$= \int_0^\infty u \frac{k^\alpha b^{k-2}\lambda\mu}{k^\alpha\lambda + b^{k-2}\mu} e^{-b^{k-2}\mu u} du.$$

$$= \frac{k^\alpha\lambda}{b^{k-2}\mu\left(k^\alpha\lambda + b^{k-2}\mu\right)}, \quad for \quad k > 2. \tag{3.13}$$

Similarly, the expected length of cold standby time can be computed as follows:

$$E\left[X_k^{(2)} - Y_k^{(1)} I_{\{X_K^{(2)} - Y_K^{(1)} > 0\}}\right] = \int_0^\infty v g(v) dv. \tag{3.14}$$

Where $g\left(v\right)$ be the p.d.f of $v = X_k^{(2)} - Y_k^{(1)}$. By definition of p.d.f and using Jacobean Transformation we have:

$$g(v) = \int_0^\infty f\left(u+v, u\right) du. \tag{3.15}$$

where $\mathrm{X}_k^{(2)} = u+v,\ \mathrm{Y}_k^{(1)} = u$ such that $v = X_k^{(2)} - Y_k^{(1)}$. (3.16)

Since $X_k^{(i)}$ *and* $Y_k^{(i)}$, for i=1, 2 are all independent and form a geometric process,

$$g(v)=\int_0^{\infty} f\left(u+v\right).f(u)du. \tag{3.17}$$

Using equations (3.16) and (3.17), we get:

$$g(v)=\frac{k^{\alpha}b^{k-2}\lambda\mu}{k^{\alpha}\lambda+b^{k-2}\mu}e^{-k^{\alpha}\lambda v}, \quad for \quad v\geq 0 \tag{3.18}$$

From equations (3.15) and (3.18), we have:

$$E\left[X_k^{(2)}-Y_k^{(1)}I_{\{X_K^{(2)}-Y_K^{(1)}>0\}}\right]=\int_0^{\infty} vg(v)dv$$

$$=\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)} \tag{3.19}$$

Using the equations (3.6), (3.8), (3.13) and (3.19) ,equation (3.3) becomes:

$$E(L)=\sum_{k=1}^{N+1}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N}\frac{\mu}{b^{k-1}}+\sum_{k=1}^{N-1}\frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda+b^{k-2}\mu\right)}+\sum_{k=1}^{N}\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)} \tag{3.20}$$

From equations (3.6) ,(3.8),(3.14),(3.21)and (3.22), we have:

$$C(N)=\frac{C_r E\left[\sum_{k=1}^{N}Y_k^{(1)}+\sum_{k=1}^{N-1}Y_k^{(2)}\right]+C-C_w E\left[\sum_{k=1}^{N+1}X_k^{(1)}+\sum_{k=1}^{N}X_k^{(2)}\right]}{E(L)}$$

$$C(N)=\frac{C_r\mu\left(\sum_{k=1}^{N-1}\frac{1}{b^{k-1}}+\sum_{k=1}^{N}\frac{1}{b^{k-1}}\right)-C_w\lambda\left(\sum_{k=1}^{N+1}\frac{1}{k^{\alpha}}+\sum_{k=1}^{N}\frac{1}{k^{\alpha}}\right)+C}{\sum_{k=1}^{N+1}\frac{\lambda}{k^{\alpha}}+\sum_{k=1}^{N}\frac{\mu}{b^{k-1}}+\sum_{k=1}^{N-1}\frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda+b^{k-2}\mu\right)}+\sum_{k=1}^{N}\frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda+b^{k-1}\mu\right)}} \tag{3.21}$$

$$C(N)=\frac{C_r\mu\left(l_3+l_4\right)-C_w\lambda\left(l_1+l_2\right)+C}{\left(\lambda l_1+\mu l_3\right)+\left(l_5+l_6\right)} \tag{3.22}$$

This is the long run average cost per unit time under policy N.

Where $l_1=\sum_{k=1}^{N+1}\frac{1}{k^{\alpha}}$ $\qquad$ $l_2=\sum_{k=1}^{N}\frac{1}{k^{\alpha}}$,

$l_3=\sum_{k=1}^{N}\frac{1}{b^{k-1}}$, $\qquad$ $l_4=\sum_{k=1}^{N-1}\frac{1}{b^{k-1}}$.

$$l_5 = \sum_{k=2}^{N-1} \frac{k^{\alpha}\lambda}{b^{k-2}\mu\left(k^{\alpha}\lambda + b^{k-2}\mu\right)} \quad , \qquad l_6 = \sum_{k=1}^{N} \frac{b^{k-1}\mu}{k^{\alpha}\lambda\left(k^{\alpha}\lambda + b^{k-1}\mu\right)} .$$

Using this C (N), the optimal replacement policy N* is determined by numerical methods such that C(N*) is minimized.The next section provides some numerical results to highlight the obtained theoretical results.

4. Numerical Results And Conclusions

For the given hypothetical values of the parameters of λ, μ, α, b, C_w, C, and C_r the values of long-run average cost per unit time are calculated from the expression (3.22) as follows:

Table 2.4.1: Values of the long-run average cost rate under policy N.

λ =10, μ=20, , α=0.45, C=3000, Cw=10 Cr=50

	b=0.85	**b=0.75**
N	**C(N)**	**C(N)**
2	86.05883	84.44675
3	82.03035	80.26599
4	80.83704	**79.32042**
5	80.76468	79.58738
6	**81.19815**	80.33242
7	81.86819	81.23691
8	82.64165	82.15073
9	83.44743	83.00255
10	84.24582	83.7614
11	85.01423	84.41778
12	85.74007	84.97379
13	86.41673	85.43738
14	87.04142	85.81917
15	87.61375	86.13043

Table 2.4.2: Values of the long-run average cost rate under policy N.

	b=0.65	**b=0.55**
N	**C(N)**	**C(N)**
2	82.54412	80.26461
3	78.09312	75.41437
4	**77.27482**	**74.59189**
5	77.69543	74.97099
6	78.49591	75.60093
7	79.33187	76.16705
8	80.07002	76.5932
9	80.67119	76.8875
10	81.13776	77.08064
11	81.4884	77.20323
12	81.74589	77.27925
13	81.93177	77.32558
14	82.06417	77.35346
15	82.15749	77.37006

Conclusions:

a) From the table 4.1 and graph 2.4.1, We can observe that the long-run average cost per unit time at the time C (6) = **81.19815** is minimum at b=0.85**.** We should replace the system at the time of 6^{th} failure.

b) From the table 4.2 and graph 2.4.2, we examine that the long-run average cost per unit time at the time C (4) = **77.27482** is minimum at b=0.65**.** We should replace the system at the time of 4^{th} failure. Thus from (a) and (b), we can conclude that the value of ‘b’ the number of failures N* and the long-run average cost per unit time are positively related.

c) From this, we can say that as ‘b’ increases number of failures increases, while ‘ α ’ decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

d) If the repairman experiences with repair then the successive repair times form a decreasing geometric process, while the consecutive working times form a an increasing alpha process. Thus this model can also be applied for an improved model.

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