IJETIE
International Journal of Emerging Technology and Innovative Engineering
Volume I, Issue 4, April 2015
ISSN: 2394 - 6598
www.ijetie.org

EFFECT OF VOLUME FRACTION OF DUST ON MHD FLOW OF A FLUID BETWEEN TWO INCLINED PARALLEL PLATES UNDER GRAVITY

Sana Ahsan, R.Khare & Ajit Paul

Department of Mathematics & Statistics

Sam Higginbottom Institute of Agriculture, Technology and Sciences

Allahabad 211007, India

Abstract: In this paper, the MHD flow of a viscous incompressible fluid between two inclined parallel plates under gravity has been considered. The effect of volume fraction of dust and other parameters associated with the flow problem has been analyzed with the help of graphs. The steady and unsteady velocities of fluid are found to increase on increasing volume fraction. It has also been observed that the fluid initial velocity (steady and unsteady) increases on increasing magnetic field parameter.

Keywords: Porous medium, MHD flow ,Incompressible fluid , viscous , gravity, volume fraction, on Newtonian factor.

Introduction: In the present paper we have considered the flow of an incompressible fluid between two inclined parallel flat plates in which uniform magnetic field is applied perpendicular to the flow.Snedden[1] (1951) studied the flow of viscous incompressible fluid down an inclined plate.Sharma[2] (1995) worked on the steady laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source and sink. The three dimensional free convective flow and the heat transfer through a porous medium have been studied by Ahmed[3] (1997). Biswal[4](2002) worked on the MHD flow between two infinite parallel plates, one oscillating while other is uniform in motion. The unsteady flow through a porous medium in a vertical channel under a transverse magnetic field has been studied by Rajeshwara[5] (2003). Kumar and Singh[6] (2008) also studied the effect of magnetic field on the motion under gravity of a viscous fluid between two inclined parallel flat plates.

In this paper we have extended the research work of Kumar and Singh(2008) by including two additional parameters volume fraction of dust and non Newtonian factors. We have derived the expression for steady and unsteady velocity of fluid and then the effect of various parameters on velocity have been analyzed with the help of graphs.

Formulation of the problem

Consider the motion of a viscous incompressible fluid in an inclined plate in the presence of magnetic field. It is assumed that the velocity of the fluid at the free surface is known. The uniform magnetic field is applied perpendicular to the parallel flat plates. Consider the motion of the fluid is slow, in this case the equations of motion are

$$(1-\varphi)\frac{\partial v*}{\partial t*} - F + \frac{1}{\rho} grad(p) = -v * \nabla^2 u * - \frac{\sigma B_0^2}{\rho} v * - \frac{1}{K} v * \quad (1)$$

Insert 1 + c^2 in the above eq.

Symbols:

v*: velocity of fluid,

p: pressure,

ρ:density,

$B_{0:}$magnetic field ,

Φ: volume fraction,

c: non Newtonian factor,

t:time,

$t_{0:}$Arbitrary time,

u*: velocity,

h: distance between two plates.

Consider the surface between two parallel plates filled with viscous liquid of density ρ,with boundaries $y = 0, y = h$

Now $F = g(sin\alpha - cos\alpha, o)$ (2)

Now consider the motion to be same in all planes parallel to xy-plane. Vector V will be of the form $(u * ,0,0)$ where u is the function of x,y and t.Let us consider that the fluid is homogenous in nature.

Then equation of continuity is

$$\frac{\partial u*}{\partial x*} = 0 \quad (3)$$

In the view of this eq.(1) will become

$$(1-\varphi)\frac{\partial u*}{\partial t*} = g\ sin\alpha + \left[-\frac{1}{\rho}\frac{\partial p}{\partial x*} + \frac{v\partial^2 u*}{\partial x*}\right] - \left(\frac{\sigma B_0}{\rho(1+c^2)} + \frac{1}{K}\right)u* \quad (4)$$

$$0 = g\cos\alpha - \frac{1}{\rho}\frac{\partial p}{\partial x*} \quad (5)$$

Now since $(gsin\alpha - \frac{1}{\rho}\frac{\partial p}{\partial x*})$ is a function of x only and using eq.(5) we can write

$$p = \rho g(x*sin\alpha - y*cos\alpha) + x*\rho X$$

Where X is constant

Using this eq.(4) becomes

$$(1-\varphi)\frac{\partial u*}{\partial t*} = g\ sin\alpha - \frac{1}{\rho}\frac{\partial p}{\partial x*} + \frac{v\partial^2 u*}{\partial y*^2} - \left(\frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{1}{K}\right)u* \quad (6)$$

$$(1-\varphi)\frac{\partial u*}{\partial t*} = -X + \frac{v\partial^2 u*}{\partial y*^2} - \left(\frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{1}{K}\right)u* \quad (7)$$

Now from finite fourier sine transform we have

$$u_s(n,t) = \int_0^t u(y,t)\sin(\frac{n\pi y}{h})dy \quad (8)$$

Boundary conditions are

$$\begin{array}{lll} u* = u*(t) & \text{at} & y* = h \\ u* = 0 & \text{at} & y* = 0 \\ u* = (0,t*) & \text{at} & y* = 0 \\ u* = (h,t*) & \text{at} & y* = h \end{array} \quad (9)$$

Multiply eq.(7) by $\sin(\frac{n\pi y}{h})$ and integrating w.r.t. y from$0 \to h$

And then using boundary conditions, we can write

$$\int_0^h \frac{\partial u*}{\partial t*}(1-\varphi)\sin(\frac{n\pi y}{h})dy = X + v\int_0^h \frac{\partial^2 u*}{\partial y*^2}\sin\frac{n\pi y*}{h}dy - \left(\frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{1}{K}\right)\int_0^h u*\ \sin\frac{n\pi y*}{h}dy \quad (10)$$

$$\frac{du_s}{dt} + \left(v\frac{n^2\pi^2}{h^2} + \frac{\sigma B_0^2}{\rho(1+c^2)} + \frac{1}{K}\right)\frac{u_s(n,t)}{(1-\varphi)} = \frac{1}{(1-\varphi)}\left[-X-(-1)^{n+1}\frac{vn\pi U(t)}{h}\right] \quad (11)$$

This is ordinary linear equation with integrating factor $e^{\int\left(v\frac{n^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)\frac{1}{1-\varphi}dt}$

Using this we can write the solution as

$$u_s = \frac{-X}{\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}} + (-1)^{n+1}\frac{vn\pi}{h(1-\varphi)}\int_{t_0}^{t} u(t)e^{-\frac{1}{(1-\varphi)}\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)(t-\tau)}d\tau$$

Then by inverse formula for Finite fourier sine transform $u(y,t) = \frac{2}{h}\sum_{n=1}^{\infty}[\frac{-X}{\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}} +$

$$(-1)^{n+1}\frac{vn\pi}{h(1-\varphi)}\int_{t_0}^{t} u(t)e^{-\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)(t-\tau)}d\tau] \times \sin\frac{n\pi y}{h} \quad (12)$$

Where t_0 is the arbitrary time .

Now for steady velocity let us consider $u(t) = A(constant)$

Putting the value of u(t) in eq.(12) and solving we get

$$u(y,t) = \frac{2}{h}\sum_{n=1}^{\infty}[\frac{-X}{\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}} + (-1)^{n+1}\frac{vn\pi}{h(1-\varphi)}Ae^{-\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)t}\{e^{\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)t} - e^{\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)t_0}\}] \times \sin\frac{n\pi y}{h} \quad (13)$$

Now for unsteady velocity let us consider $u(t) = e^{-\lambda\tau}$

$$u(y,t) = \frac{2}{h}\sum_{n=1}^{\infty}[\frac{-X}{\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}} + (-1)^{n+1}\frac{vn\pi}{h(1-\varphi)}\frac{e^{-\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}\right)t}}{\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}-\lambda\right)}\{e^{\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}-\lambda\right)t} - e^{\left(\frac{vn^2\pi^2}{h^2}+\frac{\sigma B_0^2}{\rho(1+c^2)}+\frac{1}{K}-\lambda\right)t_0}\}] \times \sin\frac{n\pi y}{h} \quad (14)$$

Equation (13) gives the expression for steady velocity of fluid and eq.(14) gives the expression for unsteady velocity of fluid. The above equations are used to find numerical values of parameters to study their effect on the velocity of fluid .The result is obtained by the variation of certain parameters ϕ,σ,k,B_0,c etc.

Equation (13) gives the expression for steady velocity of fluid and eq.(14) gives the expression for unsteady velocity of fluid. Now the graphs are plotted by assuming different values of the parameters.

Figure 1(a)

Velocity of fluid(steady) and volume fraction

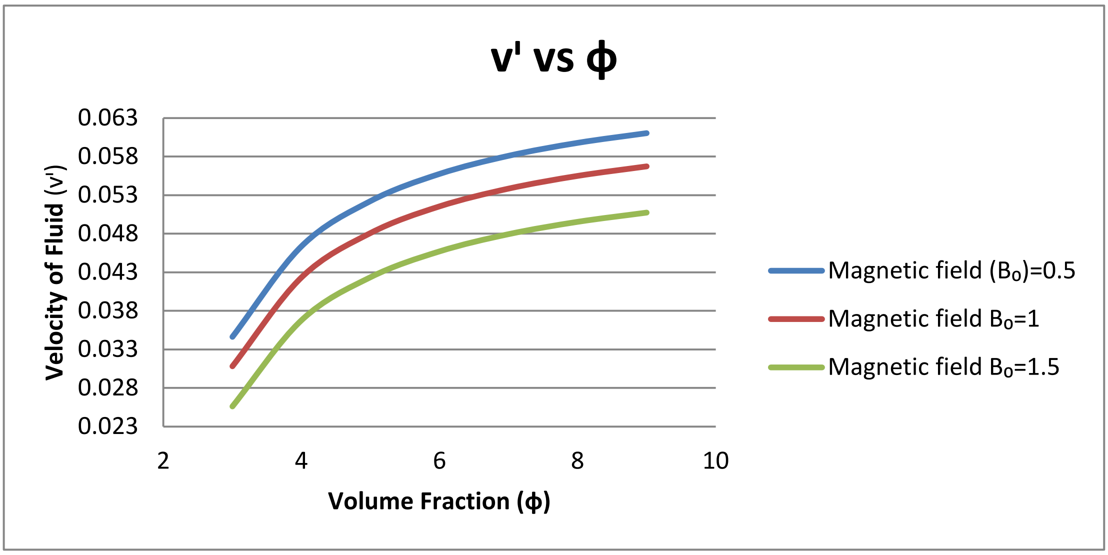
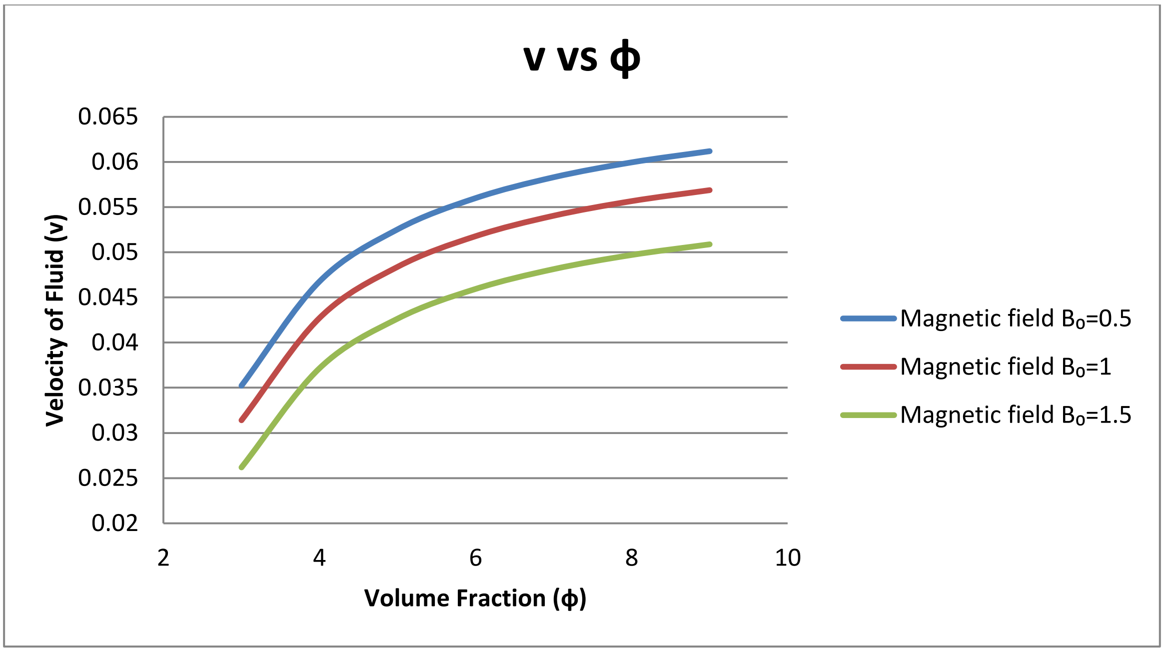


Figure 1(b)

Velocity of fluid (unsteady) and volume fraction



Result and Discussion

Figure 1(a) and 1(b)

It represents the change in steady velocity of fluid with respect to volume fraction of dust. It has been observed from the derived relation (equation13) that on increasing the volume fraction the steady velocity of the fluid is found to increase .On increasing the magnetic field, the velocity also increases. Also as the parameter of volume fractions comes in the denominator of the specified equation, so it also reduces the velocity due to the increased attraction. It is also observed that the changes in the magnetic field parameter causes specific variations in the velocity of the fluid, it is seen that the initial velocity decreases as we increase the value of magnetic field parameter. However, the effects of other parameters cannot be ruled out.

The 1(b) represents the graph depicting the change in the unsteady velocity of the fluid with respect to volume fraction of dust. It is observed that the velocity of fluid increases on increasing the value of volume fraction which is also evident from the derived equation for unsteady velocity (equation 14).It is seen that the initial velocity of the fluid also decreases on increasing the magnetic field parameter .On close observation it has been found that the effect is more prominent in case of steady velocity in comparison to unsteady velocity.

References

1.Snedden, I.N.(1951): Fourier Transform ,Mc Graw Hill Book Co.,New York.

2.Prasad V.R. and Ramacharyulu N.C.P. (1979): Unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient, Defence Science Journal, Vol. 30, pp. 125-130.

3.Debnath L. and Ghosh A.K. (1989): On unsteady hydromagnetic flows of a dusty viscous fluid between two oscillating plates, Journal of Applied Mathematics and Simulation, Vol. 2, No. 1, pp. 13-31.

4.Sharma,P.R.and Mathur, P.(1995): Steady Laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink;Indian Journal Pure Applied Mathematics,26(11), pp 1125.

5.Ahmed , N. and Sharma,D.(1997): Three dimensional free convective flow and heat transfer through a porous medium, Indian Jouranal Pure Applied Mathematics,28(10),pp.1345.

6.Chamka A.J. (1998): Unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium. International Communications in Heat and

Mass Transfer, Vol. 25, No. 6, pp. 899-906.

7.Biswal, T.,Bastia, S.N. and Patra ,P.K. (2002):MHD flow between two infinite parallel plates , one oscillating while other is uniform in motion,Acta Ciencia Indica .Vol.XXVIII M, no. 4 pp.625.

8.Shiv Prasad,R. and Rajeshwar, U.et.al(2003): Unsteady flow through a porous medium in a vertical channel under a transverse magnetic field , Acta Ciencia Indica Vol.XXIXM.no.2,pp.305.

9. Elbashbeshy E.M.A. and Bazid M.A.A. (2004) Heat transfer over an unsteady stretching surface. Heat and Mass Transfer, Vol. 41, No. 1, pp. 1-4.

10.Kumar, Agney and Singh, K.(2008): Magnetic field on the motion under gravity of a viscous fluid between two inclined parallel flat plates Acta Ciencia Indica.Vol.XXXIV,M.no.4,pp.1573.

11.**Gireesha H.M., Bagewadi C.S. and Prasannaku B.C. (2009):**Pulsatile flow of an unsteady dusty fluid through rectangular channel. Communications in Nonlinear Science and Numerical Simulation, Vol. 14, No. 5, pp. 2103-2110.

12.Khare ,R.and Singh,Neetu(2009): MHD flow of dusty viscous fluid through a circular tube . Journal of International Academy of Physical Sciences.Vol.13.pp.389-394.

13. Ezzat A.A., Bary E-1 and Morsey M.M. (2010): Space approach to the hydro-magnetic flow of a dusty fluid through a porous medium. Computers and Mathematics with Applications, Vol. 59, No. 8, pp. 2868-2879.

14.Khare ,R. and Singh, N (2010):MHD flow of a dusty viscous Incompressible fluid confined between two vertical walls with volume fraction of dust, Journal of International Academy of Physical Sciences , Vol.14 no:2 pp 215-219.(ISSN 0974-9373).

15.Singh, J.and Gupta,P.C.(2011): Effect of porous medium on MHD viscous in compressible fluid between two inclined parallel flat plates under gravity,International Journal of Mathematical Archive-2(7), pp.1180-1183.

16.Khare, R. and Rai, Abha (2011): Effect of magnetic field on the flow of dusty viscous fluid between two inclined planes. Journal of SITBS: Universal Review Vol.2,pp.123-132.

17.Khare,R. and Singh ,Jyoti (2011): Mhd flow of a dusty fluid between two inclined parallel plates . Journal of International Academy of Physical Sciences Vol. 15, pp.7-15.

18.Khare ,R. and Singh, Jyoti (2012): "Flow of an unsteady conducting dusty fluid through an inclined circular channels".Journal "Jnanabha" of Vijnana Parishad of India Vol.42 pp.31-38.

19.Khare, R. and Singh,Jyoti (2012):MHD flow of dusty non Newton fluid through an inclined Hexagonal Channel .Indian Society of Mathematics and Mathematical Sciences Gorakhpur. Vol 7 ,pp. 9-16.